1. Use the definition of the derivative to find $\frac{d}{dx}(x^2+3x)$.

We use the definition $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$:

$$\frac{d}{dx}(x^2+3x) = \lim_{h\to 0} \frac{[(x+h)^2+3(x+h)]-[x^2+3x]}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2+3x+3h-x^2-3x}{h} = \lim_{h\to 0} \frac{2xh+h^2+3h}{h}$$

$$= \lim_{h\to 0}(2x+h+3) = 2x+3.$$

2. For each of the following, find $f'(x)$ or $\frac{dy}{dx}$, as appropriate.

(a) $f(x) = x^2 \sin^{-1}(x)$

We use the product rule:

$$f'(x) = (x^2)'(\sin^{-1} x) + (x^2)(\sin^{-1} x)' = 2x \sin^{-1} x + \frac{x^2}{\sqrt{1-x^2}}.$$

(b) $y = \ln(x^2+1)$

We use the chain rule (with $u = x^2+1$, so $y = \ln u$):

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{1}{u}(u') = \frac{2x}{x^2+1}.$$

(c) $y = x^{\sin x}$

We use logarithmic differentiation. First we take logs of both sides so that we can pull the exponent down:

$$\ln(y) = \ln(x^{\sin x}) = (\sin x)(\ln x).$$

Now we differentiate both sides (implicitly on the left, product rule on the right):

$$\frac{y'}{y} = (\cos x)(\ln x) + (\sin x)(1/x).$$

Now we solve for y' and replace y with $x^{\sin x}$:

$$y' = y\Big((\cos x)(\ln x) + \frac{\sin x}{x}\Big) = x^{\sin x}\Big((\cos x)(\ln x) + \frac{\sin x}{x}\Big).$$

(d) $f(x) = \dfrac{x}{\sqrt{4x-1}}$

We use the quotient rule, with a chain rule inside it:

$$f'(x) = \frac{(x)'\sqrt{4x-1} - x\Big((4x-1)^{1/2}\Big)'}{\sqrt{4x-1}^2} = \frac{\sqrt{4x-1} - x\left(\frac{1}{2}(4x-1)^{-1/2}(4)\right)}{4x-1}$$
$$= \frac{\sqrt{4x-1} - \frac{2x}{\sqrt{4x-1}}}{4x-1} = \frac{2x-1}{(4x-1)^{3/2}}.$$

3. Find the equation for the line tangent to the graph of $y = e^{x^2}$ at the point where $x = 1$.

First note that $x = 1$ implies that $y = e^{1^2} = e$, so our point is $(1, e)$. For the slope, we differentiate (using the chain rule):

$$y' = e^{x^2}(2x) = 2xe^{x^2},$$

which, when $x = 1$, is equal to $2e$. This is the slope. Thus the line is given by

$$y - e = 2e(x-1) \qquad \Rightarrow \qquad y = 2ex - e.$$

4. Use the formula for differentiating inverses to verify that $\dfrac{d}{dx}(\tan^{-1} x) = \dfrac{1}{1+x^2}$.

If we let $f(x) = \tan x$, then this problem asks for the derivative of $g(x) = f^{-1}(x)$. Thus we have

$$g'(x) = \frac{1}{f'(g(x))} = \frac{1}{\sec^2(\tan^{-1}(x))}.$$

If we set $\theta = \tan^{-1}(x)$, then $g'(x) = \dfrac{1}{\sec^2(\theta)}$. But then $\tan(\theta) = x$, so the corresponding right triangle has opposite leg length x and adjacent leg length 1. So the hypotenuse has length $\sqrt{1+x^2}$. From this we deduce that $\sec(\theta) = \sqrt{1+x^2}$, so that

$$g'(x) = \frac{1}{\sec^2(\theta)} = \frac{1}{1+x^2}.$$