Notation: $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ and $S' = \{\mathbf{v}'_1, \mathbf{v}'_2, \mathbf{v}'_3\}$ bases for V; $T = \{\mathbf{w}_1, \mathbf{w}_2\}$ and $T' = \{\mathbf{w}'_1, \mathbf{w}'_2\}$ bases for W.

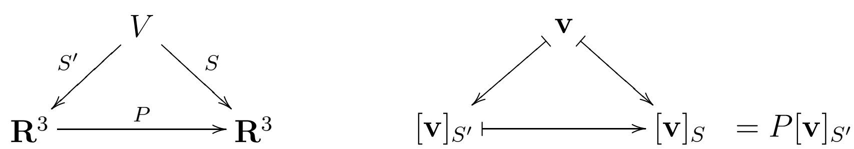
$\diamond$COORDINATES: $[\mathbf{v}]_S = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ MEANS: $\mathbf{v} = a\mathbf{v}_1 + b\mathbf{v}_2 + c\mathbf{v}_3$

$\diamond$TRANSITION MATRICES: $P_{S\leftarrow S'}$

IS: $$\begin{bmatrix} | & | & | \\ [\mathbf{v}'_1]_S & [\mathbf{v}'_2]_S & [\mathbf{v}'_3]_S \\ | & | & | \end{bmatrix}$$

DOES: $[\mathbf{v}]_S = P_{S\leftarrow S'}[\mathbf{v}]_{S'}$

PICTURE:



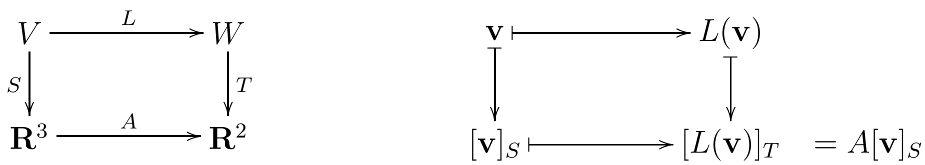
IS FOUND BY: $$\begin{bmatrix} | & | & | & | & | & | & | \\ \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 & | & \mathbf{v}'_1 & \mathbf{v}'_2 & \mathbf{v}'_3 \\ | & | & | & | & | & | & | \end{bmatrix} \rightsquigarrow \begin{bmatrix} I_3 & | & P_{S\leftarrow S'} \end{bmatrix}$$

$\diamond$MATRIX REPRESENTATIONS: A representing $L : V \to W$ w.r.t. S and T

IS: $$\begin{bmatrix} | & | & | \\ [L(\mathbf{v}_1)]_T & [L(\mathbf{v}_2)]_T & [L(\mathbf{v}_3)]_T \\ | & | & | \end{bmatrix}$$

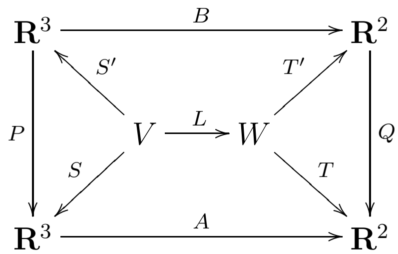
DOES: $A[\mathbf{v}]_S = [L(\mathbf{v})]_T$

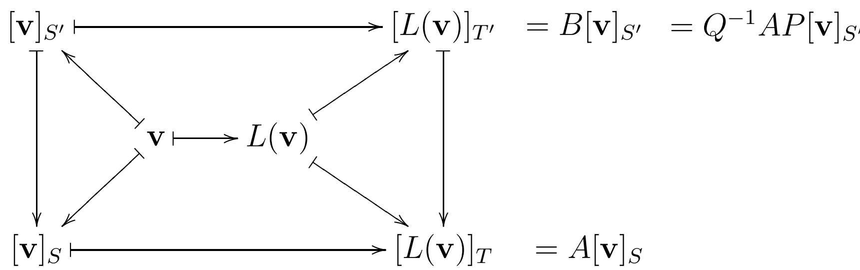
PICTURE:



IS FOUND BY: $$\begin{bmatrix} | & | & | & | & | & | \\ \mathbf{w}_1 & \mathbf{w}_2 & | & L(\mathbf{v}_1) & L(\mathbf{v}_2) & L(\mathbf{v}_3) \\ | & | & | & | & | & | \end{bmatrix} \rightsquigarrow \begin{bmatrix} I_2 & | & A \end{bmatrix}$$

⋄Relating Matrix Representations: $B = Q^{-1}AP$





Theorem. *Suppose A is an $n \times n$ matrix, and $L : V \to W$ is a linear map represented by A with respect to some bases S and T for V and W respectively. Then the following are equivalent (i.e., if one is true, all are true; if one is false, all are false):*

- *A is nonsingular/invertible*
- *L is an isomorphism*
- *$A\mathbf{x} = \mathbf{0}$ has only the trivial solution*
- *L is 1–1*
- *$A\mathbf{x} = \mathbf{b}$ has a solution for all $\mathbf{b}$*
- *L is onto*
- *A row-reduces to the identity I_n*
- *the rank of A is n*
- *the dimension of the range of L is n*
- *the nullity of A is zero*
- *the dimension of the kernel of L is zero*
- *the columns of A are linearly independent in $\mathbf{R}^n$*
- *the columns of A span $\mathbf{R}^n$*