How to do some practice problems

Section 3.3

6. (a) yes; (b) no (you can scalar multiply by a negative number to get out of the purported subspace); (c) yes; (d) no (fails both conditions)

19. Suppose $A\mathbf{v} = \mathbf{b}$. Consider $5\mathbf{v}$, for instance. Is it still a solution?

21. This was a worksheet problem.

27. You're checking to see if the three given vectors can be combined to produce a specific other vector, so line the three vectors up and augment with one of the specific vectors. Then row reduce. If you get a row of 0s in the coefficient part with something nonzero on the right, then there's a problem. Otherwise, you're fine.

Section 3.4

2. Now that we know about dimension, it's easy to see that the first two sets cannot possibly span $\mathbf{R}^4$, because there are fewer than four vectors in each. For the other two, line them up in a matrix with augmented column $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ and check that there is always a solution. Of course, not having a solution is the same as ending up with a row of 0s in the coefficient matrix part, so a simpler method involves just lining up the given vectors and checking to see if you can get a row of 0s. If you can, they don't span.

7. Row reduce (with a right-most column of 0s). Find a general solution. Pull out the arbitrary constants. The resulting vectors span the kernel.

15. Line them all up in a matrix (with a column of 0s on the right, if you like), then row reduce. If the system has a nontrivial solution (i.e., if you get to put some r into the answer) then it's dependent. Otherwise it's independent.

21. It is independent. The idea is to show that if T were dependent, S would be also. So suppose some nontrivial linear combination of vectors in T combined to form the zero vector. Take this linear combination and replace each vector in T with its expression in terms of vectors in S. Rearrange the result so as to form a linear combination of vectors in S equal to the zero vector. Show that not all the coefficients can be zero.

Section 3.5

7. (a) Because there are three vectors in the set (which is the correct number), you need only check one of span/linear independence. Either way, the problem comes down to lining up the vectors and row reducing. You should get leading 1s in all three columns and no zero rows. It follows that this is a basis.

(b) This set contains the zero vector. Any set with a zero vector cannot be independent, and so cannot be a basis.

11. Line the vectors up in a matrix, then row reduce. Once you get to row-echelon form, the columns containing the leading 1s correspond to a basis. The dimension is the number of vectors in the basis you find.

19. (a) A vector in this subspace can be written as $\begin{bmatrix} a \\ a+c \\ c \end{bmatrix} = a\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + c\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$. Thus these two vectors span the subspace. They are already linearly independent, so they form a basis.

Section 5.2

12. (a) Follows immediately from rank-nullity and the fact that $\dim\ker(L) \geq 0$.

(b) If L is onto, then the dimension of the range equals the dimension of W. Rank-nullity implies that the dimension of the range is no more than the dimension of V.

15. Note that $L(\mathbf{e}_i)$ equals the ith column of A. Theorem 5.2 says that the range of L is spanned by the $L(\mathbf{e}_i)$. The result follows.