Linear Algebra I **Worksheet 10** **Due April 17, 2007**

Name: OUID:

Instructions: Be sure to show as much work as possible, and please make a sincere effort to express your answers clearly and neatly. Please write your answers on your own paper, then staple your pages together using this sheet as a cover sheet.

1. [3 pts] Suppose λ is an eigenvalue of the $n \times n$ matrix A. Show that the set of all eigenvectors associated with λ (along with the zero vector) is a subspace of $\mathbf{R}^n$. (This is called the *eigenspace* associated to λ.)

2. (a) [3 pts] Find the characteristic equation for the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ -2 & 3 & -2 \\ 2 & 0 & 5 \end{bmatrix}$.

(b) [3 pts] Find the eigenvalues for this matrix.

(c) [3 pts] For each eigenvalue, find a basis for the corresponding eigenspace by solving the equation $A\mathbf{x} = \lambda\mathbf{x}$.

(d) [3 pts] Explain why your answers above imply that A is diagonalizable.

(e) [3 pts] Verify directly that A is diagonalizable by showing that $P^{-1}AP = D$. (In particular, find P, P^{-1}, and D.)

(f) [3 pts] The *Cayley-Hamilton theorem* says that every square matrix satisfies its own characteristic equation. Verify this theorem for A. In other words, if the characteristic polynomial is

$$p(\lambda) = \lambda^n + a_1\lambda^{n-1} + \cdots + a_{n-1}\lambda + a_n,$$

show that

$$A^n + a_1A^{n-1} + \cdots + a_{n-1}A + a_nI_n = O.$$

Bonus: Suppose $C = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ is a 2×2 symmetric matrix.

(a) [3 pts] Show that the eigenvalues of C are real by explicitly computing them.

(b) [3 pts] Show that C is diagonalizable by showing that either the eigenvalues are distinct, or C is already diagonal.