Linear Algebra I   Worksheet 2 Solutions   January 30, 2007

1. (§1.3 #12) Consider the following matrices:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 3 & -1 & 3 \\ 4 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & -2 \\ 2 & 5 \end{bmatrix} \qquad E = \begin{bmatrix} 2 & -4 & 5 \\ 0 & 1 & 4 \\ 3 & 2 & 1 \end{bmatrix} \qquad F = \begin{bmatrix} -1 & 2 \\ 0 & 4 \\ 3 & 5 \end{bmatrix}$$

If possible, compute: (a) $DA+B$ (b) EC (c) CE (d) $EB+F$ (e) $FC+D$

(a) Impossible; (b) $\begin{bmatrix} 0 & -1 & 1 \\ 12 & 5 & 17 \\ 19 & 0 & 22 \end{bmatrix}$; (c) $\begin{bmatrix} 15 & -7 & 14 \\ 23 & -5 & 29 \\ 13 & -1 & 17 \end{bmatrix}$; (d) $\begin{bmatrix} 8 & 8 \\ 14 & 13 \\ 13 & 9 \end{bmatrix}$;

(e) Impossible.

2. (§1.3 #30) Consider the following linear system:

$$\begin{cases} 2x_1 + 3x_2 - 3x_3 + x_4 + x_5 & = 7 \\ 3x_1 + 2x_3 + 3x_5 & = -2 \\ 2x_1 + 3x_2 - 4x_4 & = 3 \\ x_3 + x_4 + x_5 & = 5 \end{cases}$$

(a) Find the coefficient matrix; (b) Write the linear system in matrix form; (c) Find the augmented matrix.

(a) $\begin{bmatrix} 2 & 3 & -3 & 1 & 1 \\ 3 & 0 & 2 & 0 & 3 \\ 2 & 3 & 0 & -4 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$; (b) $\begin{bmatrix} 2 & 3 & -3 & 1 & 1 \\ 3 & 0 & 2 & 0 & 3 \\ 2 & 3 & 0 & -4 & 0 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 7 \\ -2 \\ 3 \\ 5 \end{bmatrix}$;

(c) $\left[\begin{array}{ccccc|c} 2 & 3 & -3 & 1 & 1 & 7 \\ 3 & 0 & 2 & 0 & 3 & -2 \\ 2 & 3 & 0 & -4 & 0 & 3 \\ 0 & 0 & 1 & 1 & 1 & 5 \end{array}\right]$.

3. (§1.4 #10) Find two different 2×2 matrices A such that $A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Possible answers include (among many others) $\pm \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\pm \begin{bmatrix} 1 & b \\ 0 & -1 \end{bmatrix}$, and $\pm \begin{bmatrix} 1 & 0 \\ c & -1 \end{bmatrix}$, where b and c are arbitrary.

4. (§1.4 #12) Find two different 2×2 matrices A such that $A^2 = O$.

Possible answers include (among many others) $\begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ c & 0 \end{bmatrix}$, where b and c are arbitrary.

5. (§1.6 #6) Sketch $\mathbf{u} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$ and $f(\mathbf{u})$, where $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$.

The vector $f(\mathbf{u})$ is parallel to $\mathbf{u}$ and pointing in the same direction, but twice as long.

6. (§1.6 #12) Determine whether the vector $\mathbf{w} = \begin{bmatrix} 8 \\ 5 \\ 3 \end{bmatrix}$ is in the range of the matrix transformation $f : \mathbf{R}^2 \to \mathbf{R}^3$ defined by $f(\mathbf{x}) = A\mathbf{x}$, where

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

We need to determine if $A\mathbf{x} = \mathbf{w}$ has a solution or not. This matrix equation corresponds to a system of linear equations. Row reducing the corresponding augmented matrix, we obtain:

$$\begin{bmatrix} 1 & 2 & | & 8 \\ 0 & 1 & | & 5 \\ 1 & 1 & | & 3 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 2 & | & 8 \\ 0 & 1 & | & 5 \\ 0 & 0 & | & 0 \end{bmatrix},$$

which has a solution. It follows that $\mathbf{w}$ is in fact in the range of f.

7. (§1.6 #16) Give a geometric description of the matrix transformation $f : \mathbf{R}^2 \to \mathbf{R}^2$ corresponding to the matrix $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Note that $f(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix}$, so that this map switches coordinates. It follows that this map is a reflection across the line $y = x$.