

Name:

OID:

**Instructions:** Be sure to show as much work as possible, and please make a sincere effort to express your answers clearly and neatly. Please write your answers on your own paper, then staple your pages together using this sheet as a cover sheet.

1. [12 pts] Let  $L : \mathbf{R}^4 \rightarrow \mathbf{R}^4$  be the linear transformation defined by

$$L \left( \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \right) = \begin{bmatrix} a - b \\ 0 \\ c - d \\ 0 \end{bmatrix}.$$

- (a) Show directly that  $L$  is linear.
- (b) Find the standard matrix representation for  $L$ .
- (c) Describe the kernel of  $L$ .
- (d) Describe the range of  $L$ .

2. [3 pts] Show that the set of vectors of the form  $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$ , where  $b = a + c$ , forms a subspace of  $\mathbf{R}^3$ .

3. [6 pts] Given a finite set of vectors  $S = \{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_k\}$  in  $\mathbf{R}^n$ , show that  $\text{span}(S)$  is a subspace of  $\mathbf{R}^n$ .