

Name:

OID:

**Instructions:** Be sure to show as much work as possible, and please make a sincere effort to express your answers clearly and neatly. Please write your answers on your own paper, then staple your pages together using this sheet as a cover sheet.

1. [3 pts] Do the following vectors span  $\mathbf{R}^4$  (justify your answer):

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -5 \\ 2 \end{bmatrix} \right\}$$

2. [6 pts] (a) Find a basis for the kernel of the linear map  $L : \mathbf{R}^4 \rightarrow \mathbf{R}^4$  corresponding to multiplication by the following matrix:

$$\begin{bmatrix} 1 & 1 & 2 & -1 \\ 2 & 3 & 6 & -2 \\ -2 & 1 & 2 & 2 \\ 0 & -2 & -4 & 0 \end{bmatrix}$$

- (b) Find a basis for the range of this same map.

3. [6 pts] Show that the following is a basis for  $\mathbf{R}^4$ :  $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

4. [3 pts] Find a basis for the subspace of  $\mathbf{R}^4$  consisting of vectors of the form  $\begin{bmatrix} a+c \\ a-b \\ b+c \\ -a+b \end{bmatrix}$ . What is its dimension?

5. [3 pts] Find a basis for  $\mathbf{R}^4$  that includes the vectors  $\begin{bmatrix} 2 \\ 3 \\ 6 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$ .