Linear Algebra I **Worksheet 5** **Due February 27, 2007**

Name: OUID:

Instructions: Be sure to show as much work as possible, and please make a sincere effort to express your answers clearly and neatly. Please write your answers on your own paper, then staple your pages together using this sheet as a cover sheet.

1. [9 pts] You are given the following vector and two ordered bases for $\mathbf{R}^3$:

$$S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \right\} \qquad T = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\} \qquad \mathbf{v} = \begin{bmatrix} 7 \\ -1 \\ 9 \end{bmatrix}$$

(a) Find the coordinate vectors $[\mathbf{v}]_T$ and $[\mathbf{v}]_S$ directly.

(b) Find the transition matrix $P_{S \leftarrow T}$.

(c) Verify your answers to parts (a) and (b) by checking that

$$[\mathbf{v}]_S = P_{S \leftarrow T}[\mathbf{v}]_T.$$

2. [3 pts] Let $T = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ be a basis for $\mathbf{R}^2$. Given that $P_{S \leftarrow T} = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, find the basis S for $\mathbf{R}^2$,

3. [3 pts] Let $S = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$ be a basis for $\mathbf{R}^2$. Given that $P_{S \leftarrow T} = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$, find the basis T for $\mathbf{R}^2$,

4. [6 pts] Let $L : \mathbf{R}^3 \to \mathbf{R}^3$ be defined by $L\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix} \right) = \begin{bmatrix} 2a - c \\ a + b - c \\ c \end{bmatrix}$.

(a) Find the matrix representation for L with respect to the basis $S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(b) Suppose $[\mathbf{v}]_S = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$. Use your answer to part (a) to find $[L(\mathbf{v})]_S$ and $L(\mathbf{v})$.