Linear Algebra I **Worksheet 6** **Due March 6, 2007**

Name: OUID:

Instructions: Be sure to show as much work as possible, and please make a sincere effort to express your answers clearly and neatly. Please write your answers on your own paper, then staple your pages together using this sheet as a cover sheet.

1. Consider the following bases for $\mathbf{R}^2$ and $\mathbf{R}^3$:

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\} \qquad S' = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$$

$$T = \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\} \qquad T' = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

and note that the transition matrices are given by

$$P_{S \leftarrow S'} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} \qquad Q_{T \leftarrow T'} = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}.$$

Let $L : \mathbf{R}^2 \to \mathbf{R}^3$ be a linear map whose matrix representation with respect to S and T is $A = \begin{bmatrix} -1 & -1 \\ 0 & 3 \\ 2 & -1 \end{bmatrix}$, and let $[\mathbf{v}]_{S'} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$.

(a) [3 pts] Find the inverse of Q.

(b) [3 pts] Use the matrices above and your answer to part (a) to find the matrix representation B for L with respect to S' and T'.

(c) [3 pts] Use matrix multiplication to find $[\mathbf{v}]_S$, $[L(\mathbf{v})]_T$ and $[L(\mathbf{v})]_{T'}$

(d) [3 pts] Find $\mathbf{v}$ and $L(\mathbf{v})$.

(e) [6 pts] Find the matrix representation for L with respect to the standard bases for $\mathbf{R}^2$ and $\mathbf{R}^3$.

2. [3 pts] Let V be the set of all ordered pairs (a, b) of real numbers with the operations

$$(a, b) \oplus (c, d) = (a + c, b + d) \qquad r \odot (a, b) = (a, rb).$$

Show that this set with these operations is *not* a vector space.