

Name:

OUID:

**Instructions:** Be sure to show as much work as possible, and please make a sincere effort to express your answers clearly and neatly. Please write your answers on your own paper, then staple your pages together using this sheet as a cover sheet.

1. [12 pts] Consider the subset  $S = \{t^2 + t, 2t^2 + 1, t + 2\}$  of vectors in  $P_2$ .

(a) Show directly that  $S$  is a basis.

(b) Find  $[\mathbf{v}]_S$ , where  $\mathbf{v} = t^2 + t - 5$ .

(c) Find the transition matrix  $P_{T \leftarrow S}$ , where  $T = \{t^2 + t, t + 1, 1\}$ .

(d) If  $[\mathbf{u}]_T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ , what is  $\mathbf{u}$ ?

2. [6 pts] Consider the function  $L : P_2 \rightarrow \mathbf{R}^1$  given by  $L(p(t)) = \int_{-1}^1 p(t) dt$ .

(a) Show that  $L$  is linear.

(b) Find the kernel of  $L$ . What is its dimension?

3. [3 pts] Let  $W$  be the subspace of  $M_{22}$  spanned by the following matrices:

$$\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} \right\}$$

Use an isomorphism between  $M_{22}$  and  $\mathbf{R}^4$  to find a basis for  $W$  inside this set.