Linear Algebra I **Worksheet 8** **Due April 3, 2007**

Name: OUID:

Instructions: Be sure to show as much work as possible, and please make a sincere effort to express your answers clearly and neatly. Please write your answers on your own paper, then staple your pages together using this sheet as a cover sheet.

1. Let V be the vector space of continuous functions on $[0, 1]$ and define an inner product by $\langle f, g\rangle = \int_0^1 f(t)g(t)\,dt$.

(a) Find the cosine of the angle between t^2 and t.

(b) Find the length of t^2.

(c) For what values of a and b are $3t + 1$ and $at + b$ orthogonal?

(d) Let W be the subspace of V spanned by $\{1, 2t - 1, 6t^2 - 6t + 1\}$. Note that this is an orthogonal basis for W. Find the vector in W nearest to the vector given by $f(t) = \cos(t)$.

2. Prove the parallelogram law for any two vectors in an inner product space:

$$\| \mathbf{u} + \mathbf{v} \|^2 + \| \mathbf{u} - \mathbf{v} \|^2 = 2 \| \mathbf{u} \|^2 + 2 \| \mathbf{v} \|^2 .$$

3. Let W be the subspace of $\mathbf{R}^4$ spanned by $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 1 \\ -2 \end{bmatrix}$.

(a) Find a basis for $W^\perp$.

(b) Write $\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ as the sum of a vector in W and a vector in $W^\perp$.