

Name:

OUID:

Instructions: Be sure to show as much work as possible, and please make a sincere effort to express your answers clearly and neatly. Please write your answers on your own paper, then staple your pages together using this sheet as a cover sheet.

1. [6 pts] Find the Fourier polynomial of degree 2 for $\sin^2(t)$.
2. [6 pts] Use the Gram-Schmidt process to find an orthogonal basis for the kernel of the linear map $L : \mathbf{R}^4 \rightarrow \mathbf{R}^1$ defined by

$$L \left(\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \right) = a - b - 2c + d.$$

3. [6 pts] Find the matrix associated to the inner product $\langle p(t), q(t) \rangle = \int_{-1}^1 p(t)q(t) dt$ on P_2 .
4. [3 pts] Let $L_\theta : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ be the linear map which rotates counterclockwise about the origin by an angle θ , and let A_θ be the standard 2×2 matrix representation of L_θ . For which θ is A_θ diagonalizable? Explain. (You should never have to write out the matrix A_θ explicitly for this.)