Quiz 1 Solutions

Calculus III **January 15, 2008**

Instructions: Please be sure to write neatly, show your work, and put your answer in a box.

1. Let T be the triangle in $\mathbf{R}^3$ with corners at the points $(1, 4, 0)$, $(0, -2, 1)$ and $(1, 1, 1)$.

(a) Find a vector orthogonal to the plane in which T lies.

By subtracting components of pairs of points in the triangle, we find vectors contained in the plane. So, for example the vectors

$$\langle 1-1, 4-1, 0-1\rangle = \langle 0, 3, -1\rangle$$

and

$$\langle 0-1, -2-1, 1-1\rangle = \langle -1, -3, 0\rangle$$

are contained in the plane of the triangle. To obtain an orthogonal vector, we cross these two vectors, obtaining

$$\langle 0, 3, -1\rangle \times \langle -1, -3, 0\rangle = \langle -3, 1, 3\rangle.$$

(b) Find the area of T.

The area of T is half the area of the parallelogram spanned by the two vectors found above. This in turn is equal to the length of the cross product. Thus we have

$$\text{Area}(T) = \frac{1}{2}|\langle -3, 1, 3\rangle| = \frac{1}{2}\sqrt{3^2 + 1^2 + 3^2} = \frac{\sqrt{19}}{2}.$$

(c) Is T a right triangle? Explain.

No. The dot product of the two vectors above is

$$\langle 0, 3, -1\rangle \cdot \langle -1, -3, 0\rangle = -9 \neq 0,$$

so they are no perpendicular. Also, the dot product is negative, so the angle between them is greater than ninety degrees. As they are both based at the same corner of the triangle ($(1, 1, 1)$), this implies the triangle is obtuse, and thus not right. Alternatively, you can find the vector along the third side of the triangle and check that none of the three pairs of dot products are zero.