Quiz 3

Calculus III **February 12, 2008**

1. Consider the function $f(x,y) = x^3y^2 - 2x^2y$.

(a) Compute $\dfrac{\partial f}{\partial x}$.

$$\frac{\partial f}{\partial x} = 3x^2y^2 - 4xy$$

(b) Compute $\dfrac{\partial f}{\partial x}(1,3)$.

$$\frac{\partial f}{\partial x}(1,3) = 3(1)(3^2) - 4(1)(3) = 27 - 12 = 15$$

(c) Find the equation for the y-trace of the surface $z = f(x,y)$ where $y = 3$.

$$f(x,3) = 9x^3 - 6x^2$$

(d) Differentiate the equation found in part (c) and evaluate at $x = 1$.

$$\frac{d}{dx} f(x,3)\bigg|_{x=1} = (27x^2 - 12x)\big|_{x=1} = 27 - 12 = 15$$

(e) Verify Clairaut's theorem for the original function $f(x,y)$; i.e., show that $f_{xy} = f_{yx}$.

$$f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}(3x^2y^2 - 4xy) = 6x^2y - 4x$$
$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}(2x^3y - 2x^2) = 6x^2y - 4x$$