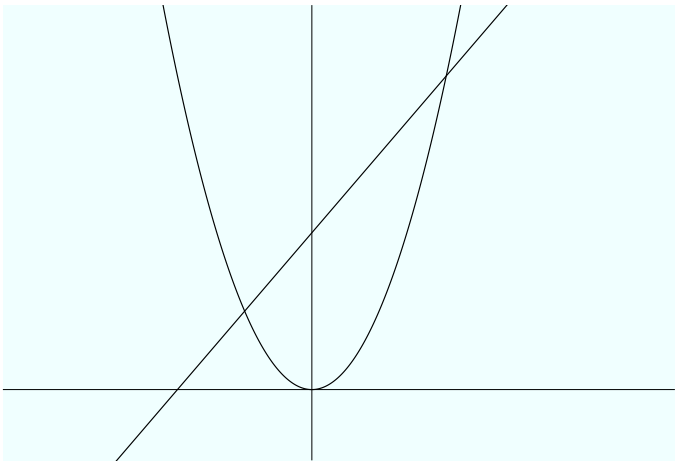
Quiz 5

Name: **Calculus III**

ID: **March 4, 2008**

1. Compute $\iint_D x\,dA$, where D is the region in the xy-plane bounded by the curve $y = x^2$ and the line $y = x + 2$, shown below.



This region is x-simple, so the limits of integration for y are $x^2 \le y \le x + 2$. To find the x-limits, we need to find the intersection points of the two curves. For this we have

$$x^2 = x + 2 \quad \Rightarrow \quad x^2 - x - 2 = (x+1)(x-2) = 0 \quad \Rightarrow \quad x = -1, 2.$$

These then are the x-limits. Now we compute

$$\iint_D x\,dA = \int_{-1}^{2}\int_{x^2}^{x+2} x\,dy\,dx = \int_{-1}^{2} [xy]_{y=x^2}^{y=x+2}\,dx$$

$$= \int_{-1}^{2} \left[x(x+2) - x(x^2)\right]\,dx = \int_{-1}^{2} (x^2 + 2x - x^3)\,dx$$

$$= \frac{x^3}{3} + x^2 - \frac{x^4}{4}\bigg|_{x=-1}^{x=2} = \left(\frac{8}{3} + 4 - 4\right) - \left(\frac{-1}{3} + 1 - \frac{1}{4}\right) = \frac{9}{4}.$$