Quiz 10 Solutions Calculus II April 24, 2009

1. Convert the polar point $(-4, \frac{\pi}{3})$ to Cartesian (rectangular) coordinates. Also give two other sets of polar coordinates for the same point, at least one of which has a positive r-coordinate.

For Cartesian coordinates, we compute

$$(x, y) = (r\cos\theta, r\sin\theta) = (-4(1/2), -4(\sqrt{3}/2)) = (-2, -2\sqrt{3}).$$

For other polar coordinates, we can add even multiples of π to the angle, leaving r unchanged, or we may add odd multiples of π to the angle and change the sign of r. For example, we have

$$(-4, \frac{\pi}{3}) = (-4, \frac{7\pi}{3}) = (4, \frac{4\pi}{3}).$$

2. Find the length of the equiangular spiral $r = e^\theta$ for $0 \le \theta \le 2\pi$.

Using the polar arc-length formula, along with the facts that $(e^\theta)' = e^\theta$ and $(e^\theta)^2 = e^{2\theta}$, we have

$$L = \int_0^{2\pi} \sqrt{e^{2\theta} + e^{2\theta}}\, d\theta = \sqrt{2}\int_0^{2\pi} e^\theta\, d\theta = \sqrt{2}(e^{2\pi} - e^0) = \sqrt{2}(e^{2\pi} - 1).$$