Quiz 6 Solution Calculus II

1. Find the volume of the solid whose base is the region above the x-axis and inside the circle $x^2 + y^2 = 4$, where the cross-sections perpendicular to the x-axis are right triangles with height equal to half the base. (The object looks like a wedge cut out of a tree.) It may be helpful to sketch and label both the solid and a typical cross-section.

The area of a cross-section is $\frac{1}{2}bh$. Since the height is half the base, this becomes $\frac{1}{4}b^2$. We need now to express b in terms of x. If you draw the semicircular base of the solid, you see that, because the triangular slices are perpendicular to the x-axis, then b is exactly the y-coordinate of the corresponding point on the circle $x^2 + y^2 = 4$. Thus $b = y = \sqrt{4 - x^2}$. Finally, we're integrating from -2 to 2. Thus we have

$$V = \frac{1}{4}\int_{-2}^{2}(4-x^2)\,dx = \frac{1}{4}\left(4x - \frac{x^3}{3}\right)\Big|_{-2}^{2} = \frac{1}{4}\left[\left(8 - \frac{8}{3}\right) - \left(-8 - \frac{-8}{3}\right)\right] = \frac{8}{3}.$$