

1. Let R denote the region shown on the board, bounded above by the line $y = 1$, below by the curve $y = \sqrt{x}$, and to the left by the y -axis. A tank is formed by rotating this region around the y -axis. (Units are in meters.)

(a) Find the volume of a horizontal slab of the tank of thickness Δy .

$$V = \pi r^2 \Delta y = \pi x^2 \Delta y = \pi (y^2)^2 \Delta y = \pi y^4 \Delta y.$$

(b) Find the volume of the tank by integrating.

$$V = \int_0^1 \pi y^4 dy = \frac{\pi y^5}{5} \Big|_0^1 = \frac{\pi}{5}.$$

(c) Suppose the tank is filled with water. Find the work done in pumping the water out through a vertical spout 1 meter long at the top of the tank.

For a slice we have

$$W = Fd = mgd = V\rho gd = (\pi y^4 \Delta y)(1000)(9.8)(2 - y),$$

so for the whole thing we have

$$W = 9800\pi \int_0^1 y^4(2 - y) dy = 9800\pi \int_0^1 (2y^4 - y^5) dy = \frac{6860\pi}{3}.$$