Quizzes 8 & 9 Solutions Calculus II

1. The density of deer in a forest is the radial function $\rho(r) = \dfrac{150}{(r^2+2)^2}$ deer per square kilometer, where r is the distance (in kilometers) to a small meadow. Calculate the number of deer in the region $2 \leq r \leq 5$ km.

The area of a thin washer-shaped region is approximately $2\pi r \Delta r$, so the number of deer in this area is approximately $\rho(r) 2\pi r \Delta r$. Thus the total number of deer is

$$\int_2^5 2\pi r \frac{150}{(r^2+2)^2}\, dr = \frac{-150\pi}{r^2+2}\Big|_2^5 = \frac{175\pi}{9},$$

where we've used the substitution $u = r^2 + 2$, $du = 2r\, dr$.

2. Find the length of the graph of the portion of the line $y = mx + r$ between $x = a$ and $x = b$ in two ways (hint: you should get the same answer for both):

(a) Using the arclength integral formula $\displaystyle\int_a^b \sqrt{1 + [f'(x)]^2}\, dx$;

Note that $\frac{dy}{dx} = m$, so we have

$$L = \int_a^b \sqrt{1+m^2}\, dx = \sqrt{1+m^2}\, x|_a^b = (b-a)\sqrt{1+m^2}.$$

(b) Using the Pythagorean theorem and the picture below.

$$L = \sqrt{(b-a)^2 + m^2(b-a)^2} = \sqrt{(1+m^2)(b-a)^2} = (b-a)\sqrt{1+m^2}.$$