Answer to even homework problems from week 2

3.4 #12: (a) Dependent: $3\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3 = \mathbf{v}_4$.

(b) Independent.

(c) Independent.

3.4 #14: (a) Independent.

(b) Independent.

(c) Independent.

(d) Dependent: $\cos 2t = \cos^2 t - \sin^2 t$.

3.4 #16: Dependent when $c = 1$.

3.4 #22: Dependent.

3.4 #28: Independent.

3.5 #2: Only (c) is a basis.

3.5 #4: Bases are (c) and (d).

3.5 #8: (a) Not a basis.

(b) This is a basis, and $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} + 2 \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} - 1 \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix}$.

3.5 #10: (a) This is a basis, and $5t^2 - 3t + 8 = -3(t^2 + t) + 8(t^2 + 1)$.

(b) Not a basis.

3.5 #12: A basis is given by the first three vectors, and the dimension is three.

3.5 #14: The first two vectors are a basis.

3.5 #16: One answer is

$$\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right\}.$$

3.5 #18: The first two form a basis, and the dimension is two.

3.5 #20: (a) One answer is $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$.

(b) One answer is $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \\ 1 \end{bmatrix} \right\}$.

(c) One answer is $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix} \right\}$.

3.5 #22: One answer is $\{t^3 + 5t^2 - 13t,\ 3t^2 - 9t + 1\}$.

3.5 #28: (a) One answer is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$.

(b) One answer is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$.

3.5 #24: (a) The dimension is three.

(b) The dimension is two.

3.5 #26: (a) Two

(b) Three

(c) Three

(d) Three

3.5 #30: One answer is $\left\{ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$. The dimension is six. In general the dimension of M_{mn} is mn.

3.5 #32: Two

3.5 #34: P_1, for example.

1.3 #12: (a) Not possible

(b) $\begin{bmatrix} 0 & 1 & 1 \\ 12 & 5 & 17 \\ 19 & 0 & 22 \end{bmatrix}$

(c) $\begin{bmatrix} 15 & -7 & 14 \\ 23 & -5 & 29 \\ 13 & -1 & 17 \end{bmatrix}$

(d) $\begin{bmatrix} 8 & 8 \\ 14 & 13 \\ 13 & 9 \end{bmatrix}$

(e) Not possible

1.3 #14: (a) $\begin{bmatrix} 58 & 12 \\ 66 & 13 \end{bmatrix}$

(b) $\begin{bmatrix} 58 & 12 \\ 66 & 13 \end{bmatrix}$

(c) $\begin{bmatrix} 28 & 8 & 38 \\ 34 & 4 & 41 \end{bmatrix}$

(d) $\begin{bmatrix} 28 & 8 & 38 \\ 34 & 4 & 41 \end{bmatrix}$

(e) $\begin{bmatrix} 28 & 32 \\ 16 & 18 \end{bmatrix}$ for both

(f) $\begin{bmatrix} -16 & -8 & -26 \\ -30 & 0 & -31 \end{bmatrix}$

3.7 #2: $\begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix}$

3.7 #4: $\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$

3.7 #6: $\begin{bmatrix} -1 \\ 2 \\ -2 \\ 4 \end{bmatrix}$

3.7 #8: $\mathbf{v} = \begin{bmatrix} 3 & 1 & 3 \end{bmatrix}$

3.7 #10: $\mathbf{v} = t^2 - 3t + 2$

3.7 #12: $\mathbf{v} = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$

3.7 #14: (a) $[\mathbf{v}]_T = \begin{bmatrix} -9 \\ -8 \\ 28 \end{bmatrix}$, $[\mathbf{w}]_T = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} -2 & -5 & -2 \\ -1 & -6 & -2 \\ 1 & 2 & 1 \end{bmatrix}$

(c) $[\mathbf{v}]_S = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $[\mathbf{w}]_S = \begin{bmatrix} -18 \\ -17 \\ 8 \end{bmatrix}$

(e) $\begin{bmatrix} -2 & 1 & -2 \\ -1 & 0 & -2 \\ 4 & -1 & 7 \end{bmatrix}$

3.7 #16:

3.7 #18: $[\mathbf{v}]_S = \begin{bmatrix} 5 \\ 3 \end{bmatrix}$

3.7 #20: $[\mathbf{v}]_T = \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}$

3.7 #22: $T = \left\{ \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix} \right\}$

3.7 #24: $T = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix} \right\}$