Answers to even homework problems from week 3

1.4 #10: $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ work, for instance.

1.4 #12: $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ work, for instance.

1.4 #30: $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = B, C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ works, for instance.

1.4 #32: $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ works, for instance.

1.6 #10: No.

1.6 #12: Yes.

1.6 #14: No.

1.6 #16: (a) Switches coordinates (reflects across the line $y = x$).

(b) Switches coordinates and changes signs (reflects across the line $y = -x$).

1.6 #18: (a) $\mathbf{u} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ work, for instance.

(b) $\mathbf{u} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ work, for instance.

5.1 #2: (a) No.

(b) No.

(c) Yes.

5.1 #26: No; it doesn't send the zero vector to the zero vector, for instance.

5.1 #28: $L(\begin{bmatrix} a & b & c & d \end{bmatrix}) = \begin{bmatrix} -2a - 5b + 3c + 4d & 14a + 19b - 12c - 14d \end{bmatrix}$.

3.6 #2: (a) Solutions are vectors that look like $\begin{bmatrix} 2a \\ 2b \\ a+b \end{bmatrix}$.

(b) $\begin{bmatrix} 2a \\ 2b \\ a+b \end{bmatrix} = a \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + b \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$.

3.6 #4: One basis is $\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \\ 6 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$. The dimension is three.

3.6 #6: One basis is $\left\{ \begin{bmatrix} 1 \\ 1 \\ -2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ -2 \\ 1 \\ 0 \end{bmatrix} \right\}$. The dimension is two.

3.6 #8: The solution space is the zero vector, with dimension zero.

3.6 #10: One basis is $\left\{ \begin{bmatrix} 17 \\ 0 \\ 5 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\}$. The dimension is two.

3.6 #12:

5.1 #6: (a) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$

5.1 #8: (a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

5.1 #10: (a) $\begin{bmatrix} 5 \\ -4 \\ -7 \end{bmatrix}$

(b) $\begin{bmatrix} -u_2 + 2u_3 \\ -2u_1 + u_2 + 3u_3 \\ u_1 + 2u_2 - 3u_3 \end{bmatrix}$

5.1 #12: (a) $L(\begin{bmatrix} -1 & 5 \end{bmatrix}) = \begin{bmatrix} 8 & 5 \end{bmatrix}$.

(b) $L(\begin{bmatrix} u_1 & u_2 \end{bmatrix}) = \begin{bmatrix} \frac{3}{2}u_2 - \frac{1}{2}u_1 & -\frac{5}{2}u_1 + \frac{1}{2}u_2 \end{bmatrix}$.

5.1 #18: $17t - 7$

5.1 #34: It's the identity matrix.

5.2 #2: (a) No; (b) Yes; (c) Yes; (d) No; (e) $\begin{bmatrix} -2r \\ r \end{bmatrix}$; (f) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$.

5.2 #4: (a) $\{\begin{bmatrix}0 & 0\end{bmatrix}\}$; (b) Yes; (c) No.

5.2 #6: (a) $\{1\}$ with dimension one; (b) $\{2t^3, t^2\}$ with dimension two.

5.2 #8: (a) $\{t^2 - t + 1\}$; (b) $\{t, t+1\}$.

5.2 #10: (a) $\left\{\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \begin{bmatrix}0 & 2\\1 & 0\end{bmatrix}\right\}$; (b) $\left\{\begin{bmatrix}0 & -2\\1 & 0\end{bmatrix}, \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}\right\}$.

5.2 #16:

5.2 #22:

3.8 #2: $\{t^3 + t^2 + 2t + 1, t^3 - 3t + 1, t^2 + t + 2, t + 1\}$

3.8 #4: $\{\begin{bmatrix}1 & 2\end{bmatrix}, \begin{bmatrix}2 & 3\end{bmatrix}\}$

3.8 #6:

3.8 #8:

3.8 #10: (a) Both are two; (b) Both are two.

3.8 #18: (a) singular; (b) nonsingular.

3.8 #20: (a) yes; (b) no.

3.8 #24: Yes.

3.8 #26: Yes.

3.8 #28: (a) Three.

3.8 #30: (a) Zero, one, two, or three; (b) Three; (c) Two.

5.2 #26: (a) Seven; (b) Five.