Quiz 2 July 17, 2006 Name:

1. Find an invertible matrix P and a diagonal matrix D so that $P^{-1}AP = D$, where

$$A = \begin{bmatrix} 2 & 0 & 0 \\ -4 & -4 & 2 \\ -8 & -12 & 6 \end{bmatrix}$$

(You do ***not*** have to invert P or multiply anything out; just find P and D.)

For the eigenvalues we calculate

$$\det \begin{bmatrix} 2-\lambda & 0 & 0 \\ -4 & -4-\lambda & 2 \\ -8 & -12 & 6-\lambda \end{bmatrix} = (2-\lambda)[(-4-\lambda)(6-\lambda)+24] = -\lambda(\lambda-2)^2.$$

It follows that

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

To find P we find the corresponding eigenvectors. For $\lambda = 0$ we have

$$\left[\begin{array}{ccc|c} 2 & 0 & 0 & 0 \\ -4 & -4 & 2 & 0 \\ -8 & -12 & 6 & 0 \end{array}\right] \mapsto \left[\begin{array}{ccc|c} 2 & 0 & 0 & 0 \\ 0 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right].$$

The general solution to this is

$$\begin{bmatrix} 0 \\ r \\ 2r \end{bmatrix} = r \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}.$$

For $\lambda = 2$ we have

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ -4 & -6 & 2 & 0 \\ -8 & -12 & 4 & 0 \end{array}\right] \mapsto \left[\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ -4 & -6 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right],$$

the general solution to which is

$$\begin{bmatrix} r \\ s \\ 2r+3s \end{bmatrix} = r \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}.$$

It follows that

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 2 & 3 \end{bmatrix}.$$