Week 1 worksheet
(due Monday, June 12 at the beginning of class)

1. Let V be the set of all 2×1 matrices $\begin{bmatrix} x \\ y \end{bmatrix}$ with operations defined as follows:

$$\begin{bmatrix} p \\ q \end{bmatrix} \oplus \begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} p+s \\ q+t \end{bmatrix} \qquad c \odot \begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} cs \\ 0 \end{bmatrix}.$$

Is V a vector space? If so, prove it. If not, list any properties of Definition 3.4 that fail to hold.

2. Let $\mathbf{Z}$ denote the set of all integers $\ldots, -2, -1, 0, 1, 2, \ldots$. Is $\mathbf{Z}$ a subspace of the set of all real numbers $\mathbf{R}$ (with the usual operations)? If so, prove it. If not, why not?

3. Let $V = C(-1, 1)$ be the set of all real-valued functions that are continuous between -1 and 1 (but not necessarily at the endpoints). Let W be the subset of V consisting of those functions with the further property that

$$\int_{-1}^{1} f(x)\, dx = 0.$$

Is W a subspace of V? If so, prove it. If not, why not?

4. (a) Let $S = \{t^2 + 2t + 1,\ t^2 + 3,\ t - 1\}$ be a set of vectors in P_2. Is $t^2 - 1$ in the span of S?

(b) Find a relationship that must be satisfied by a, b, c in order for $at^2 + bt + c$ to lie in the span of S.

5. (a) Show that the set

$$S = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \right\}$$

spans M_{22}.

(b) Write $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ as a linear combination of the matrices in S.