Week 2 worksheet
(due Monday, June 19 at the beginning of class)

1. (a) Show that the following set of vectors in M_{22} is linearly dependent:

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} 3 & -4 \\ 1 & 5 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}.$$

(b) Express one vector as a linear combination of the others.

2. Find a basis for $\mathbf{R}^4$ containing the following vectors:

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}.$$

3. Find a spanning set for the subspace of P_2 consisting of polynomials $at^2 + bt + c$, where $a + 2b = c$.

4. Let $S = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$ and $T = \left\{ \begin{bmatrix} 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \end{bmatrix} \right\}$ be ordered bases for $\mathbf{R}^2$. Let $\mathbf{v} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$.

(a) Find the coordinate vector for $\mathbf{v}$ with respect to the basis T.

(b) Find the transition matrix P from the T-basis to the S-basis.

(c) Use your answer to (b) to find the coordinate vector for $\mathbf{v}$ with respect to S.

5. (a) Let $S = \{\mathbf{v}_1, \mathbf{v}_2\}$ and $T = \{t + 2, 1\}$ be ordered bases for P_1. Find the basis S using the fact that the transition matrix from S to T is $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$.

(b) Let $S = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$ and $T = \{\mathbf{w}_1, \mathbf{w}_2\}$ be ordered bases for $\mathbf{R}^2$. Find the basis T, using the fact that the transition matrix from S to T is $\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$.