

Week 3 worksheet
(due Tuesday, June 27 at the beginning of class)

1. Let $L : P_1 \rightarrow P_2$ be a linear transformation for which we know that

$$L(t + 1) = t^2 + t - 2 \quad \text{and} \quad L(t - 1) = t^2 - t - 1.$$

Find $L(6t - 4)$.

2. (a) Find the standard matrix representing the linear transformation $L : \mathbf{R}^3 \rightarrow \mathbf{R}^4$ where

$$L \left(\begin{bmatrix} a \\ b \\ c \end{bmatrix} \right) = \begin{bmatrix} a - b \\ a + c \\ b + c \\ b - a \end{bmatrix}.$$

- (b) Find a set of vectors spanning the kernel of L .
(c) Find a set of vectors spanning the image of L .
3. Define a linear transformation $L : M_{22} \rightarrow P_2$ by

$$L \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = (a + b + d)t^2 + (b - c)t + (c - d).$$

- (a) Find a set of vectors spanning the kernel of L .
(b) Find a set of vectors spanning the image of L .
4. Verify the matrix version of the rank-nullity theorem for the following matrix:

$$A = \begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 1 & 2 & 2 & 1 & 2 \\ 2 & 4 & 3 & 3 & 3 \\ 0 & 0 & 1 & -1 & -1 \end{bmatrix}.$$

5. Let $L : V \rightarrow \mathbf{R}^5$ be a linear transformation.
- (a) If L is one-to-one and $\dim(\text{Im } L) = 3$, what is $\dim(V)$?
(b) If $\dim(V) = 3$ and $\dim(\ker L) = 1$, what is $\dim(\text{Im } L)$?
(c) If L is onto, what can we say about $\dim(V)$?
(d) If L is one-to-one, what can we say about $\dim(V)$?
(e) If L is one-to-one and onto, what can we say about $\dim(V)$?