Week 6 worksheet
(due Monday, July 17 at the beginning of class)

1. Find the eigenvalues and corresponding eigenvectors for the matrix $\begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$.

2. The matrix A below has $\lambda = 2$ as an eigenvalue of multiplicity two. Determine if this eigenvalue is defective or not.

$$A = \begin{bmatrix} 3 & 2 & 1 & 1 \\ 1 & 2 & 1 & -1 \\ 2 & 3 & 4 & 1 \\ 0 & 1 & 0 & 3 \end{bmatrix}$$

3. Suppose there are four teams in a curling league. At the end of the season, the results are as follows:

Team 1 beat teams 2 and 3, but lost to team 4.
Team 2 beat team 3, but lost to teams 1 and 4.
Team 3 beat team 4, but lost to teams 1 and 2.
Team 4 beat teams 1 and 2, but lost to team 3.

(a) Form the corresponding matrix A that reflects these results, where

$$a_{ij} = \begin{cases} 1 & \text{if team } i \text{ beat team } j \\ 0 & \text{otherwise} \end{cases}$$

(b) How small can the dominant eigenvalue for A be? How large? Explain.

(c) It turns out that the dominant eigenvalue is approximately 1.395, and the corresponding eigenvector is $\mathbf{v} = \begin{bmatrix} 0.552 \\ 0.321 \\ 0.448 \\ 0.626 \end{bmatrix}$. How should the teams be ranked?

4. Is the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ diagonalizable? Explain.

5. Find an invertible matrix P and a diagonal matrix D so that $P^{-1}AP = D$, where

$$A = \begin{bmatrix} 4 & 2 \\ 3 & 3 \end{bmatrix}.$$