Week 7 worksheet
(due Monday, July 24 at the beginning of class)

1. Consider the vector space $\mathbf{R}^2$ endowed with the inner product given by

$$\left\langle \begin{bmatrix} a \\ b \end{bmatrix}, \begin{bmatrix} c \\ d \end{bmatrix} \right\rangle = ac - ad - bc + 5bd.$$

(a) Find the lengths of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

(b) Find the cosine of the angle between $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

2. Consider the vector space $C[0,1]$ of functions continuous between zero and one, endowed with the inner product $\langle f, g \rangle = \int_0^1 f(t)g(t)\,dt$.

(a) Find the length of $f(t) = e^t$.

(b) Find all values for a and b so that $g(t) = at + b$ is orthogonal to $f(t) = t + 1$.

3. Let W be the subspace of $\mathbf{R}^4$ spanned by $\begin{bmatrix} 2 \\ 0 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -6 \\ 1 \\ 5 \\ -8 \end{bmatrix}$. Find a basis for $W^\perp$.

4. In $C[0,1]$, find the projection of $t + 1$ onto t^2.

5. Let W be the subspace of $\mathbf{R}^3$ spanned by $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$.

(a) Find the distance between $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ and the nearest vector in W.

(b) Write $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ as $\mathbf{w} + \mathbf{u}$, where $\mathbf{w}$ is in W and $\mathbf{u}$ is in $W^\perp$.