Coordinates, Transition Matrices, and Matrix Representations

Notation: To simplify things, we'll assume V is 3–dimensional and W is 2–dimensional, just so we can be more explicit in the notation. Of course, all of this generalizes in obvious ways to spaces of any finite dimension.

We let $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ and $S' = \{\mathbf{v}'_1, \mathbf{v}'_2, \mathbf{v}'_3\}$ be bases for V, and $T = \{\mathbf{w}_1, \mathbf{w}_2\}$ and $T' = \{\mathbf{w}'_1, \mathbf{w}'_2\}$ be bases for W.

Remark: On this handout, the notation $[\mathbf{v}]$ (i.e., looks like coordinates but without any subscript specifying a basis) means the coordinates with respect to the standard basis of V (or W, as the case may be). In particular, if $V = \mathbf{R}^n$, then $[\mathbf{v}]$ means just the original column vector $\mathbf{v}$ itself.

◇COORDINATE VECTOR: $[\mathbf{v}]_S = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

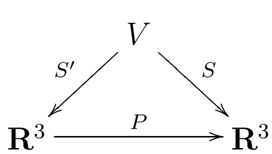
MEANS: $\mathbf{v} = a\mathbf{v}_1 + b\mathbf{v}_2 + c\mathbf{v}_3$

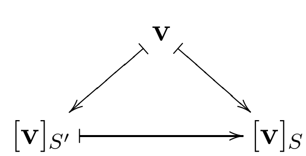
◇TRANSITION MATRIX: $P_{S \leftarrow S'}$

IS: $\begin{bmatrix} | & | & | \\ [\mathbf{v}'_1]_S & [\mathbf{v}'_2]_S & [\mathbf{v}'_3]_S \\ | & | & | \end{bmatrix}$

DOES: $[\mathbf{v}]_S = P_{S \leftarrow S'}[\mathbf{v}]_{S'}$

PICTURE:





$= P[\mathbf{v}]_{S'}$

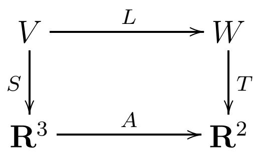
IS FOUND BY: $\left[\begin{array}{ccc|ccc} | & | & | & | & | & | \\ [\mathbf{v}_1] & [\mathbf{v}_2] & [\mathbf{v}_3] & [\mathbf{v}'_1] & [\mathbf{v}'_2] & [\mathbf{v}'_3] \\ | & | & | & | & | & | \end{array}\right] \rightsquigarrow \left[\begin{array}{c|c} I_3 & P_{S \leftarrow S'} \end{array}\right]$

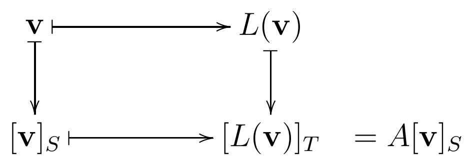
$\diamond$Matrix Representation: A representing $L: V \to W$ w.r.t. S and T

IS: $$\begin{bmatrix} | & | & | \\ [L(\mathbf{v}_1)]_T & [L(\mathbf{v}_2)]_T & [L(\mathbf{v}_3)]_T \\ | & | & | \end{bmatrix}$$

DOES: $A[\mathbf{v}]_S = [L(\mathbf{v})]_T$

PICTURE:





IS FOUND BY: $$\left[\begin{array}{cc|ccc} | & | & | & | & | \\ [\mathbf{w}_1] & [\mathbf{w}_2] & [L(\mathbf{v}_1)] & [L(\mathbf{v}_2)] & [L(\mathbf{v}_3)] \\ | & | & | & | & | \end{array}\right] \rightsquigarrow \left[\begin{array}{c|c} I_2 & A \end{array}\right]$$

$\diamond$Relating Matrix Representations: $B = Q^{-1}AP$