

Name:

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**Instructions:** Be sure to show as much work as possible, and please make a sincere effort to express your answers clearly and neatly. Please write your answers on your own paper, then staple your pages together using this sheet as a cover sheet.

1. Consider the linear map  $L : \mathbf{R}^4 \rightarrow \mathbf{R}^4$  given by  $L \left( \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \right) = \begin{bmatrix} b - d \\ a + b + c \\ -a + 4d \\ 3b + c - d \end{bmatrix}$ .

(a) [3 pts] Find the standard matrix representation  $A$  for  $L$ .

(b) [4 pts] Show that  $L$  is an isomorphism by finding the inverse of the matrix  $A$  from part (a).

(c) [3 pts] Use your answer to part (b) to write a formula for  $L^{-1} : \mathbf{R}^4 \rightarrow \mathbf{R}^4$  like that for  $L$  above.

2. Consider the basis  $S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$  for  $\mathbf{R}^3$ .

(a) [3 pts] Suppose  $[\mathbf{v}]_S = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ . Find  $\mathbf{v}$ .

(b) [3 pts] Find the  $S$ -coordinates for  $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ .

3. [5 pts] Show that  $[\mathbf{v}]_S + [\mathbf{w}]_S = [\mathbf{v} + \mathbf{w}]_S$  for any two vectors  $\mathbf{v}$  and  $\mathbf{w}$  and any basis  $S$ .