Linear Algebra I **Worksheet 4** **Due June 27, 2007**

Name: OUID:

Instructions: Be sure to show as much work as possible, and please make a sincere effort to express your answers clearly and neatly. Please write your answers on your own paper, then staple your pages together using this sheet as a cover sheet.

1. Consider the following bases for $\mathbf{R}^3$:

$$T = \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\} \qquad T' = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

(a) Find the transition matrix $Q_{T \leftarrow T'}$.

(b) Find the inverse of Q.

2. Given the basis $S = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$ for $\mathbf{R}^2$, suppose $S' = \{\mathbf{v}_1, \mathbf{v}_2\}$ is another basis for $\mathbf{R}^2$, so that the transition matrix $P_{S \leftarrow S'}$ is $\begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$.

(a) Find the basis S'.

(b) Suppose $[\mathbf{v}]_{S'} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$. Use matrix multiplication to find $[\mathbf{v}]_S$.

3. Let $L : \mathbf{R}^2 \to \mathbf{R}^3$ be defined by $L\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} -a \\ a - b \\ a \end{bmatrix}$.

(a) Find the matrix representation for L with respect to S and T, given above.

(b) Use matrix multiplication to find $[L(\mathbf{v})]_T$, where $\mathbf{v}$ is as above.

(c) Use matrix multiplication to find $[L(\mathbf{v})]_{T'}$.