Linear Algebra I **Worksheet 5** **Due July 6, 2007**

Name: OUID:

Instructions: Be sure to show as much work as possible, and please make a sincere effort to express your answers clearly and neatly. Please write your answers on your own paper, then staple your pages together using this sheet as a cover sheet.

1. Suppose V is the set of all positive real numbers, and define operations $\oplus$ and $\odot$ as follows:

- if $\mathbf{u}$ is the positive real number a, and $\mathbf{v}$ is the positive real number b, then define $\mathbf{u} \oplus \mathbf{v}$ to be ab (so $\oplus$ is actually multiplication)
- if $\mathbf{u}$ is the positive real number a, then define $\alpha \odot \mathbf{u}$ to be a^α (so $\odot$ is actually exponentiation)

Show that V with these operations is a vector space, by showing that it is closed under $\oplus$ and $\odot$, and showing that it satisfies properties 1–8 in the definition of a real vector space.

2. (a) Show that the set of all continuous functions $f(x)$ with the property that $f(3) = 0$ is a subspace of $C(-\infty, \infty)$.

(b) Show that the set of all continuous functions $f(x)$ with the property that $f(0) = 3$ is not a subspace of $C(-\infty, \infty)$.

3. (a) Do the matrices $\left\{ \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ -5 & 2 \end{bmatrix} \right\}$ span M_{22}?

(b) Are the polynomials $\{t^3 + t^2, t + 1, t^3 + 1, t^2 + t + 1\}$ linearly independent in P_3?

4. Let W be the subspace of P_2 spanned by

$$S = \{t^2 + 2t + 2,\ 3t^2 + 2t + 1,\ 11t^2 + 10t + 7,\ 7t^2 + 6t + 4\}.$$

(a) Find the coordinates of these vectors with respect to the basis $\{t^2+t+1, t^2+t, t^2\}$.

(b) Use the coordinate vectors found in part (a) to find a basis for W consisting of vectors from S.