

Name:

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Instructions: Be sure to show as much work as possible, and please make a sincere effort to express your answers clearly and neatly. Please write your answers on your own paper, then staple your pages together using this sheet as a cover sheet.

1. Compute e^A , where $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$.

2. Suppose there are four teams in a league. At the end of the season, the results are as follows:

Team 1 beat teams 2 and 3, but lost to team 4.

Team 2 beat team 3, but lost to teams 1 and 2.

Team 3 beat team 4, but lost to teams 1 and 2.

Team 4 beat teams 1 and 2, but lost to team 3.

(a) Form the corresponding matrix A that reflects these results.

(b) How small can the dominant eigenvalue for A be? How large? Explain.

(c) It turns out that the dominant eigenvalue is approximately 1.395, and the corre-

sponding eigenvector is $\mathbf{v} = \begin{bmatrix} 0.552 \\ 0.321 \\ 0.448 \\ 0.626 \end{bmatrix}$. How should the teams be ranked?

3. Find a diagonal matrix D and an orthogonal matrix P so that $D = P^T A P$, where

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 0 & -2 & 0 \\ -2 & 0 & 3 \end{bmatrix}.$$