Linear Algebra I **Worksheet 8** **Not Due**

1. Consider the inner product space $\mathbf{R}^2$ with $\left\langle \begin{bmatrix} a \\ b \end{bmatrix}, \begin{bmatrix} c \\ d \end{bmatrix} \right\rangle = ac - ad - bc + 3bd$.

(a) Find the length of $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

(b) Find the angle between $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(c) Find the projection of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ onto $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

2. Consider the inner product space P_2 with $\langle p(t), q(t) \rangle = \int_0^1 p(t)q(t)\,dt$.

(a) Let $W = \text{span}\{1, t\}$. Find an orthonormal basis for $W^{\perp}$.

(Hint: To do this, we will need to project onto W, so first you need to orthogonalize the given basis for W; then you can proceed as we did in class.)

(b) Find the matrix associated to this inner product.

3. Find the degree 2 Fourier polynomial for $f(t) = \sin^2(t)$.

4. Use properties of inner produces to verify the parallelogram law:

$$2 \parallel \mathbf{v} \parallel^2 + 2 \parallel \mathbf{w} \parallel^2 = \parallel \mathbf{v} + \mathbf{w} \parallel^2 + \parallel \mathbf{v} - \mathbf{w} \parallel^2$$