

1. Consider the function $y = x^x$. Use logarithmic differentiation to find the first and second derivatives of y .

For the first derivative, we use logarithmic differentiation. We take logs of both sides, and then use log rules to bring down the exponent:

$$\ln y = \ln x^x = x \ln x.$$

Now we differentiate both sides:

$$\frac{1}{y} \frac{dy}{dx} = (x)' \ln x + x(\ln x)' = \ln x + 1.$$

Solving for $\frac{dy}{dx}$ gives

$$\frac{dy}{dx} = y(\ln x + 1) = x^x(\ln x + 1).$$

Now for the second derivative, we differentiate the first derivative, using the product rule:

$$\frac{d^2y}{dx^2} = (x^x)'(\ln x + 1) + x^x(\ln x + 1)'$$

We computed $(x^x)'$ above, so we have

$$\frac{d^2y}{dx^2} = (x^x(\ln x + 1))(\ln x + 1) + x^x(1/x) = x^x(\ln x + 1)^2 + x^{x-1}.$$