Calculus Mini-test 1 Solutions June 6, 2008

1. Find a parametrization of the line segment between the points $(1,3,2)$ and $(4,-1,0)$.

We need a direction vector, for which we may choose $\mathbf{v} = \langle 4-1, -1-3, 0-2\rangle = \langle 3,-4,-2\rangle$. The tail of this vector is the point $(1,3,2)$, so we need to use that as our point. Thus we have

$$\mathbf{r}(t) = \langle 1,3,2\rangle + t\langle 3,-4,-2\rangle = \langle 1+3t, 3-4t, 2-2t\rangle,$$

with $0 \le t \le 1$.

2. (a) Parametrize the surface which is the graph of the function $\rho = 2\cos\phi$.

We use the spherical identities to compute

$$\langle x,y,z\rangle = \langle \rho\cos\theta\sin\phi, \rho\sin\theta\sin\phi, \rho\cos\phi\rangle = \langle 2\cos\phi\cos\theta\sin\phi, 2\cos\phi\sin\theta\sin\phi, 2\cos\phi\cos\phi\rangle,$$

where the second equality comes from substituting in the given equation. Thus a parametrization is

$$\mathbf{r}(\phi,\theta) = \langle 2\cos\phi\cos\theta\sin\phi, 2\cos\phi\sin\theta\sin\phi, 2\cos^2\phi\rangle.$$

(b) Parametrize the surface which is the graph of the function $y = z^2 - x^2$.

Here we simply have

$$\langle x,y,z\rangle = \langle x, z^2-x^2, z\rangle,$$

so that a parametrization is given by

$$\mathbf{r}(x,z) = \langle x, z^2-x^2, z\rangle.$$

3. Match the equations (a)–(h) below with their graphs I–VIII . For those equations of the form $z = f(-,-)$, also match to each its contour map (i)–(v).

(a) $z = \dfrac{4}{1+r^2}$, VIII, (iii)

(b) $z = (x^2-y^2)^2$, V, (i)

(c) $z = x - y^2$, II, (iv)

(d) $z = xy$, I, (ii)

(e) $z = \sin^2(x) + \frac{1}{4}y^2$, III, (v)

(f) $\mathbf{r}(u,v) = \langle \cos(v), \sin(v), u\rangle$, IV

(g) $\mathbf{r}(u,v) = \langle u\cos(v), u\sin(v), v\rangle$, VI

(h) $\mathbf{r}(u,v) = \langle u\cos(v), u\sin(v), u\rangle$, VII