1. Use integration to compute the volume of the solid shown below.

   The horizontal slices shown in the figure are rectangles. The area of a rectangle is $A = \ell w$. If we let ℓ be the distance from the front face to the back face, then note that $\ell = 4$ no matter where we slice the figure. Thus we have $A = 4w$. If we let z be the vertical axis, with $z = 0$ at the base of the figure, then the front face of the figure shows two similar triangles. The larger one has height 2 and base 6, while the smaller one has height $2 - z$ and base w. Using equal ratios and solving for w, we find that $w = 3(2-z)$. Thus the area of the slice at height z is $A = 4w = 12(2 - z)$. Integrating, we find the volume:

$$\int_0^2 12(2-z)\,dz = 24.$$

2. Compute $\int x^3(x^2-1)^{3/2}\,dx$

   We use the substitution $u = x^2 - 1$, so that $du = 2x\,dx$, or $\frac{1}{2}du = x\,dx$. Plugging in du, we get

$$\int x^3(x^2-1)^{3/2}\,dx = \frac{1}{2}\int x^2(x^2-1)^{3/2}\,du.$$

   Normally we don't write this sort of intermediate step (with mixed variables), but I'm including it in order to point out that the du includes one of the x factors, leaving us with only an x^2 in front. Now we replace what remains with an expression in u, noting that $x^2 = u + 1$:

$$= \frac{1}{2}\int (u+1)u^{3/2}\,du = \frac{1}{2}\int (u^{5/2} + u^{3/2})\,du.$$

   Now that we have been able to distribute, we have an expression we can integrate directly:

$$= \frac{1}{2}\left(\frac{2}{7}u^{7/2} + \frac{2}{5}u^{5/2}\right) = \frac{1}{7}(x^2-1)^{7/2} + \frac{1}{5}(x^2-1)^{5/2}.$$

3. Compute $\int \tan x\,dx$

$$\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = -\int \frac{du}{u} = -\ln|\cos(x)| = \ln|\sec(x)|.$$