

1. Compute the following using the fundamental theorem of calculus:

$$\int_1^4 \frac{(x-2)^2}{\sqrt{x}} dx$$

We first do some algebra to the integrand:

$$\frac{(x-2)^2}{\sqrt{x}} = (x^2 - 4x + 4)x^{-1/2} = x^{3/2} - 4x^{1/2} + 4x^{-1/2}.$$

This is an expression we can integrate, obtaining

$$\int \frac{(x-2)^2}{\sqrt{x}} dx = \int (x^{3/2} - 4x^{1/2} + 4x^{-1/2}) dx = \frac{2}{5}x^{5/2} - \frac{8}{3}x^{3/2} + 8x^{1/2}.$$

Evaluating from $x=1$ to $x=4$, we have

$$\left(\frac{2}{5}(4)^{5/2} - \frac{8}{3}(4)^{3/2} + 8(4)^{1/2} \right) - \left(\frac{2}{5}(1)^{5/2} - \frac{8}{3}(1)^{3/2} + 8(1)^{1/2} \right) = \frac{26}{15}.$$