

$$1. \int x^3 e^{x^2} dx =$$

We let  $w = x^2$ , so that  $dw = 2x dx$ . Substituting this in gives

$$\frac{1}{2} \int w e^w dw.$$

We now apply integration by parts, with  $u = w$  and  $dv = e^w dw$ . The formula gives

$$\frac{1}{2} \left( w e^w - \int e^w dw \right) = \frac{1}{2} (w e^w - e^w) = \frac{1}{2} (x^2 e^{x^2} - e^{x^2}).$$

$$2. \int \cos^3 x dx =$$

We use a trig identity to obtain

$$\int \cos x (1 - \sin^2 x) dx,$$

and then use substitution with  $u = \sin x$ ,  $du = \cos x dx$ , to obtain

$$\int (1 - u^2) du = u - \frac{u^3}{3} = \sin x - \frac{1}{3} \sin^3 x.$$

$$3. \int \frac{dx}{x^2 \sqrt{x^2 - 9}} =$$

We use the trig substitution  $x = 3 \sec \theta$ , so that  $dx = 3 \sec \theta \tan \theta d\theta$ . Note then that  $x^2 - 9 = 9 \sec^2 \theta - 9 = 9 \tan^2 \theta$ . Thus we have

$$\int \frac{3 \sec \theta \tan \theta}{(9 \sec^2 \theta)(3 \tan \theta)} d\theta = \int \frac{1}{9 \sec \theta} d\theta = \frac{1}{9} \int \cos \theta d\theta = \frac{1}{9} \sin \theta.$$

Now because  $\sec \theta = x/3$ , we use a right triangle to deduce that

$$\frac{1}{9} \sin \theta = \frac{\sqrt{x^2 - 9}}{9x}.$$

$$4. \int_0^1 \frac{x^2 - \sqrt{x}}{2x} dx =$$

We use some algebra to get

$$\int \left( \frac{x^2}{2x} - \frac{\sqrt{x}}{2x} \right) dx = \frac{1}{2} \int (x - x^{-1/2}) dx = \frac{1}{2} \left( \frac{x^2}{2} - 2x^{1/2} \right) = \frac{x^2}{4} - \sqrt{x}.$$

Now we need to evaluate from  $x=0$  to  $x=1$ , and there's nothing in the indefinite integral we've obtained to suggest that we shouldn't just plug in. The thing is, the original integrand is not defined at  $x=0$ , even after simplifying the algebra (the problem is the  $x^{-1/2}$  term). So technically this should be an improper integral.

Thus we have

$$\left( \frac{x^2}{4} - \sqrt{x} \right)_{x=a}^{x=1} = \left[ \left( \frac{1}{4} - 1 \right) - \left( \frac{a^2}{4} - \sqrt{a} \right) \right] \rightarrow -\frac{3}{4},$$

where the limit is taken as  $a \rightarrow 0^+$ .

5. Write the form of the partial fraction decomposition for the following expression. Do not compute the constants.

$$\frac{x^2 - 2x + 3}{(x-2)(x^2+5)(x+1)^2} =$$

$$\frac{A}{x-2} + \frac{Bx+C}{x^2+5} + \frac{D}{x+1} + \frac{E}{(x+1)^2}$$

6. Let  $R$  be the region bounded by the curves  $y = x^2$  and  $y = x^3$ . Compute the volume of the solid obtained by rotating  $R$  about the line  $y = 1$ .

We use vertical slices, with  $R = 1 - x^3$  and  $r = 1 - x^2$ . Thus the area of a washer slice is

$$\pi(R^2 - r^2) = \pi[(1 - x^3)^2 - (1 - x^2)^2] = \pi(-2x^3 + x^6 + 2x^2 - x^4).$$

Integrating, we obtain

$$\pi \int_0^1 (-2x^3 + x^6 + 2x^2 - x^4) dx = \pi \left( -\frac{x^4}{2} + \frac{x^7}{7} + \frac{2x^3}{3} - \frac{x^5}{5} \right)_{x=0}^{x=1} = \frac{23\pi}{210}.$$

7. Compute the integral  $\int \cos^2 x \, dx$  using integration by parts, with  $u = \cos x$  and  $dv = \cos x \, dx$ , and then applying the Pythagorean identity to the resulting integral.

Doing parts as instructed, we obtain

$$\begin{aligned}\int \cos^2 x \, dx &= \sin x \cos x + \int \sin^2 x \, dx = \sin x \cos x + \int (1 - \cos^2 x) \, dx \\ &= \sin x \cos x + \int dx - \int \cos^2 x \, dx = \sin x \cos x + x - \int \cos^2 x \, dx.\end{aligned}$$

We've now gone in a circle, so we move the integral from the right over to the left, obtaining

$$2 \int \cos^2 x \, dx = \sin x \cos x + x \quad \Rightarrow \quad \int \cos^2 x \, dx = \frac{1}{2}(\sin x \cos x + x).$$