2009 Southern Regional Algebra Conference

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Unique Decompositions into Ideals for Reduced Commutative Noetherian Rings, Part I

We say that a commutative ring R has the unique decomposition into ideals (UDI) property if, for any R-module which decomposes into a finite direct sum of indecomposable ideals, this decomposition is unique up to the order and isomorphism class of the ideals. In a 2001 paper, Goeters and Olberding characterize the UDI property for Noetherian integral domains. In a joint paper with Lee Klingler, we characterize the UDI property for Noetherian reduced rings.

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Nilary and Primary Rings

Recall that if R is a commutative ring and Q is an ideal of R, then Q is said to be a primary ideal of R if $ab \in Q$ implies either $a \in Q$ or $b^n \in Q$ for some positive integer n, where $a, b \in R$. These ideals are used in the following famous result of E. Noether:

Let R be a commutative ring with ACC on ideals. If I is an ideal of R then I is a finite intersection of primary ideals.

This result can be viewed as a generalization of the Fundamental Theorem of Arithmetic. In this talk, we will discuss various generalizations of the concept of primary and of E. Noether's result to noncommutative rings.

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$\mathbb{G}$-Planar Groups

For a group G with generating set $S = \{s_1, s_2, \cdots, s_k\}$, the $\mathbb{G}$-graph of G, denoted $\Gamma(G, S)$, is the graph whose vertices are distinct cosets of $\langle s_i \rangle$ in G. Two distinct vertices are joined by an edge when the set intersection of the cosets is nonempty. A group G is $\mathbb{G}$-planar if there exists a generating set S such that the graph, $\Gamma(G, S)$, is a planar graph. In this paper, we classify the $\mathbb{G}$-planar groups.

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Functors

For any ring R, let $\mathcal{J}(R)$ be the unique Boolean lattice of two sided ideals which is isomorphic to to the lattice of natural classes of non-singular right R-modules $\mathcal{N}_f(R)$. Let $1 = 1_R = 1_Q \in R \subset Q$ be rings with $R \subset Q$ an essential extension of right R-modules. Under some appropriate assumptions it is shown that there is an isomorphism of Boolean lattices $\Psi : \mathcal{J}(R) \longrightarrow \mathcal{J}(Q)$. The natural inclusion map $\phi : R \longrightarrow Q$, induces a natural order preserving map $\phi^* : \mathcal{N}_f(Q) \longrightarrow \mathcal{N}_f(R)$ of the Boolean lattices of natural classes of Q and R. It is shown that ϕ is essentially the inverse of Ψ.

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Algebraic Representations of Abstract Partial Algebras

In this paper, an algebraic representation theory of abstract partial algebras by the members of $\mathcal{L}(\mathcal{D},\mathcal{X})$, where $\mathcal{X}$ is a linear space over $\mathbb{C}$ (the complex numbers), $\mathcal{D}$ is a subspace of $\mathcal{X}$ and $\mathcal{L}(\mathcal{D},\mathcal{X})$ is the set of all $\mathcal{X}$-valued linear maps with domain $\mathcal{D}$, is developed. In particular, necessary and sufficient conditions for an algebraic representation of an abstract partial algebra to be irreducible (resp. completely reducible) are established and aspects of the decomposition theory of completely reducible representations are developed.

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K_1-Theory and Nielsen Equivalence Classes of Free Abelianized Extensions of Groups

Let G be a finitely generated group and let $d = d(G)$ denote the minimal number of generators of G. Let F_n denote the free group of rank $n \geq d$ and let $R \hookrightarrow F_n \overset{\theta}{\twoheadrightarrow} G$ be a presentation of G. We discuss the Nielsen equivalence classes of groups of the form F_n/R', i.e. of the *free abelianized extensions* of G. Our main results provide information about these equivalence classes in terms of the abelian groups $K_1(\mathbb{Z}G) = GL(\mathbb{Z}G)/E(\mathbb{Z}G)$ that arise in algebraic K-theory.

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Commuting Probability in Rings

Motivated by a similar question from statistical group theory we ask the the following question: *What is the probability, cp(R), that two randomly chosen elements a and b of a finite or more generally compact Hausdorff topological Lie ring R commute?* It is well known (and not difficult to show) that for a non-abelian group the probability that two group elements commute is $\leqslant \frac{5}{8}$. The same holds for non-commutative rings. Exactly for which rings is commuting probability equal to $\frac{5}{8}$? What can we say if $\mathrm{cp}(R) < \frac{5}{8}$? In this case, $\mathrm{cp}(R) \leqslant \frac{9}{16}$, but can we do better? We study commuting probability for semisimple algebras in more detail. The notion of communing probability is generalized to Lie rings which, in fact, makes things simpler.

This work has been done jointly with Mihail Ursul (Oradea).

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Lower Bounds for the Number of Conjugacy Classes of a Finite Group

Let G be a finite group, and let $k(G)$ denote the number of conjugacy classes of G. It is an old problem to find lower bounds for $k(G)$ in terms of the order of G. We present some new results on this topic, thereby improving some earlier results of L. Pyber.

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In a joint paper with Basak Ay, we characterize the UDI property for Noetherian reduced rings.

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Representation Theory and Higher Algebraic K-Theory
The aim of this talk is to discuss some of the themes in my new book "Representation Theory and Higher Algebraic K-Theory" published by Chapman and Hall in 2007. Thus, I will indicate how to do representation theory of groups (e.g. finite, profinite, algebraic groups, compact Lie groups) in some nice categories, e.g. exact categories, leading to the introduction of equivariant higher algebraic K-theory as well as computations of higher K-theory of group rings.

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On Fibonacci Representations
Let A be $\{1, 2, 3, 5, 8, 13, ...\}$ the set of Fibonacci numbers and $A_0 = 0 \cup A$. In this note, we investigate several counting functions on A and A_0. It can be shown that the set A_0 is a thin set that it does not form a (additive) basis of finite order. We further study the behavior of the set nA_0 (i.e. $A_0 + A_0 + ... + A_0$ n times).

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Functions that are Compatible with Group Actions
We call a function on a group G compatible if it maps left cosets into left cosets for every subgroup of G. Clearly constant functions and left translations by group elements are compatible. If these are the only compatible functions on G, then we say that the regular action of G is affine complete. This notion comes from universal algebra.

We present some families of groups with affine complete regular actions, like for example, non-abelian groups that are generated by involutions. In general, whether a group action is affine complete is not determined by the subgroup lattice alone. Still we can show that groups with certain subgroup lattices – including all distributive lattices – do not have an affine complete action. Joint work with Andras Pongracz, ELTE, Budapest.

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Rings Determined by Cyclic Covers of Groups
For a finite group G and a cover $\mathcal{C}$ of G by cyclic subgroups, we investigate the structure of the associated ring $\mathcal{R}(\mathcal{C})$. General results are obtained and these are then applied to obtain specific results on several classes of groups.

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Restrictions of Representations of ${}^3D_4(q)$ to Proper Subgroups
We prove that the restriction of any absolutely irreducible representation of the Steinberg triality groups ${}^3D_4(q)$ in characteristic co-prime to q to any proper subgroup is reducible. This work is motivated by the problem of describing all maximal subgroups of finite classical groups.

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On Realizations of Superconformal Algebras

Superconformal algebras are supergeneralizations of the Virasoro algebra. They play an important role in string theory, conformal field theory and mirror symmetry.

Well-known examples of superconformal algebras are the N = 2 and the Big N = 4 superconformal algebras. In 1997 V. Kac and C.-J. Cheng discovered a new exceptional N = 6 superconformal algebra.

A remarkable property of these superconformal algebras is that they have realizations as Lie subalgebras of pseudodifferential symbols and associated "small" representations. This allows to obtain their realizations in matrices over a Weyl algebra. A general construction of such matrix realizations is based on spin representations of the orthogonal Lie algebras.

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Solvable PST-Groups, Mutually Permutable Products, Strong Sylow Bases, and Semipermutability

A subgroup H of a group G permutes with the subgroup K of G if $HK = KH$. A subgroup H of a group G is S-permutable in G if H permutes with all Sylow subgroups of G while H is semipermutable in G if H permutes with all subgroups K of G for which $(|H|, |K|) = 1$. We say subgroups H and K of G are mutually permutable provided every subgroup of H permutes with K and every subgroup of K permutes with H. A group is called a PST-group if S-permutability is a transitive relation. In this talk we will explore some recent results and connections between solvable PST-groups, mutually pemutable products, Sylow bases whose members are pairwise mutually permutable, and groups in which semipermutability is a transitive relation.

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On Commutative Clean Rings

A commutative ring with identity is called clean if each element is the sum of an idempotent and a unit. These are precisely the commutative exchange rings.

We discuss recent joint work with W. Burgess and give applications to rings of the form $C(X)$ where X is a Tychonoff space.

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On Strongly Graded Rings and Graded Modules

Let G be a group with indentity e. A ring R is said to be G-graded if there exist additive subgroups R_g of R such that $R_g R_h \in R_{gh}$ for all $g, h \in G$. The G-graded ring R is denoted by (R, G). We denote by supp (R, G) the support of G which is defined as $\{g \in G : R_g \neq 0\}$. The elements of R_g are called homogeneous of degree g. For $x \in R$, x can be written uniquely as $\sum_{g \in G} x_g$ where x_g is the component of x in R_g. Also, we write $h(R) = \cup_{g \in G} R_g$. Many studies in group graded rings assume R to be a strongly graded ring, i.e., $R_g R_h = R_{gh}$ for all $g, h \in G$. But this strong condition is hard to satisfy. In 1995 Refai defined three successively stronger properties that a grading may have and he investigated the relationship between these strong gradings and the stronger nondegenerate and faithful properties which are motivated by the work of Cohen and Rowen.

We introduce some results concerning graded rings and graded modules and define new types of strongly graded rings and strongly graded modules. Also we study some relations between these rings and the graded modules defined on them and give some applications. A survey of my contribution to this field, will also be given.

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An Extension of the Theory of Perverse Coherent Sheaves and Some Applications
Let G be an algebraic group acting on a variety X. Deligne and Bezrukavnikov have introduced the category of equivariant perverse coherent sheaves on X. They have shown that under strong conditions on the G-variety–for example, adjacent orbits must have dimensions differing by at least two–, there exists an intersection cohomology (IC) functor which extends a coherent sheaf on a subvariety to a perverse coherent sheaf on its closure. I will describe joint work with Achar in which we show that an IC functor can be defined in much greater generality and even in the nonequivariant setting. I will discuss two geometric applications of this theory, one to a geometric construction called S_2 extension and the other to the finite Macaulayfication problem. Time-permitting, I will conclude by explaining how these methods can be used to address a conjecture of Lusztig on special pieces in the unipotent variety.

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The Entries in the LR-Tableau
Littlewood-Richardson tableaux provide an isomorphism invariant for embeddings of a subgroup in a finite abelian p-group. We give an interpretation for each coefficient in the tableau in terms of homomorphisms in the category of embeddings.

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A Comparison of Zero Divisor Graphs
A zero divisor graph of a ring R is a visual representation of the zero divisors and their relationships in R. They have been studied by Beck, Anderson & Livingston, Mulay, and Wickham, to name just a few. Our aim, using ideas of Mulay, is to identify ring theoretic properties from these graphs. In particular, we are interested in the graph of equivalence classes of zero divisors. We will compare and contrast this graph with the original zero divisor graph and discuss some results involving the associated primes of the ring.

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Groups with a Bell Condition on Infinite Subsets
If $\mathcal{V}$ is a variety of groups defined by the law $\omega(x_1, \ldots, x_n) = 1$, a group G is said to be a $\mathcal{V}^*$*-group* if for any infinite subsets $X_1, \ldots, X_n$ of G there exist $x_1 \in X_1, \ldots, x_n \in X_n$ such that $\omega(x_1, \ldots, x_n) = 1$. Clearly $\mathcal{V} \cup \mathcal{F} \subseteq \mathcal{V}^*$, where $\mathcal{F}$ is the class of all finite groups. It is known that for many varieties $\mathcal{V}$ and for many words ω the equality $\mathcal{V}^* = \mathcal{V} \cup \mathcal{F}$ holds. However, it is still an open question whether this is true for all varieties $\mathcal{V}$ and for all words ω. Here, we consider the variety $\mathcal{B}_n$ of n-Bell groups defined by the law $[x^n, y][x, y^n]^{-1} = 1$, and we deal with some conditions under which the equality $\mathcal{B}_n^* = \mathcal{B}_n \cup \mathcal{F}$ holds. Joint work with C. Delizia - Università di Salerno.

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The Semigroup of Right Ideals of a Semigroup

This talk is based on joint work with Henry Heatherly. Let S be a semigroup. We denote the set of right ideals of S by $\mathbb{R}(S)$. The set $\mathbb{R}(S)$ forms a semigroup under right ideal multiplication. We show how conditions on $\mathbb{R}(S)$ affect the structure of S, and conversely.

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On Orbits of Automorphism Groups II

Let G be a finite group, A be a group of automorphisms of G, and let $C_G(A)$ denote the subgroup of fixed points of A in G. If the order of $C_G(A)$ is coprime to the number of orbits of A in G, then $C_G(A)$ is contained in the autocommutator subgroup $[G, A]$. The notion of class-avoiding automorphism is used to extend some results of J. Thompson and P. Rowley.

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Noetherian Skew Inverse Power Series Rings

We study skew inverse power series extensions $R[[y^{-1}; \tau, \delta]]$, where R is a noetherian ring equipped with an automorphism τ and a τ-derivation δ. We find that these extensions share many of the well known features of commutative power series rings. As an application of our analysis, we see that the iterated skew inverse power series rings corresponding to nth Weyl algebras are complete local, noetherian, Auslander regular domains whose right Krull dimension, global dimension, and classical Krull dimension, are all equal to $2n$. Joint work with Edward Letzter, Temple University.

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The Galois Extension with a Galois Commutator Subring

Let B be a Galois extension of B^G with Galois group G, Δ the commutator subring of B^G in B, and $G|_\Delta$ the restriction of G to Δ. Equivalent conditions are given for a Galois extension Δ of Δ^G with Galois group $G|_\Delta$. It is shown that the following statements are equivalent: (1) Δ is a Galois extension of Δ^G with Galois group $G|_\Delta$. (2) Let $N = \{g \in G \,|\, g(x) = x \text{ for all } x \in \Delta\}$. Then $B^N = B^G \cdot \Delta$ and Δ is a finitely generated and projective right module over Δ^G. (3) B is a composition of two Galois extensions: $B \supset B^G \cdot \Delta$ with Galois group N and $B^G \cdot \Delta \supset B^G$ with Galois group G/N such that Δ is a finitely generated and projective right module over Δ^G. Consequently, more results can be derived for several well known classes of Galois extensions such as DeMeyer-Kanzaki Galois extensions, Azumaya Galois extensions, center Galois extensions, commutator Galois extensions, and Hirata separable Galois extensions. Joint work with George Szeto.