

**Math 311 Homework 1 Fall 09 due August 27**

1. Use the Binomial Theorem to verify the following

(a) 
$$\sum_{k=0}^n \binom{n}{k} = 2^n.$$

(b) 
$$\sum_{k=0}^n (-1)^k \binom{n}{k} = 0. \text{ Assume that } n \geq 1$$

2. Use the Binomial Theorem to verify the following

$$\sum_{k=0}^n \binom{n}{k} 2^{n-2k} = \left(\frac{5}{2}\right)^n.$$

3. Use proof by induction to show that

$$\sum_{k=1}^n \frac{1}{k^n} \leq 2 - \frac{1}{n}.$$

4. Use proof by induction to show that

$$\sum_{k=1}^n \frac{k}{2^k} = 2 - \frac{n+2}{2^n}.$$

5. Use proof by induction to show that

$$\sum_{k=1}^n \binom{k+1}{2} = \frac{n(n+1)(n+2)}{6}.$$