Math 311 Fall 2009 Homework 5 due October 20.

1. Find the smallest positive integer x with
$x \equiv 3 \pmod{13}$, $x \equiv 1 \pmod{8}$ and $x \equiv 2 \pmod{7}$.

2. Professor Carroll tries to divide his class into three equal groups, but two students do not have a group. When he tries to divide them into five groups, one student is left. Finally, a configuration of seven equal groups leaves one group with one student less than the others. What is the smallest possible number of students in Professor Caroll's class?

3. I sent out 120 invitations to a singles mixer. I participated in all of the activities. We did a little mixer game where we paired off into groups of 5, but one group only had 2. Then, we had a sit down dinner with 9 to a table, but three tables ended up with 10. At the end of the evening, everyone left in pairs, except for me. How many people (including me) attended the event (assuming no uninvited guests showed up)?

4. Problem 4 on page 92.

5. (a) Problem 12 on page 93.

(b) Let p be a prime and $0 < k < p$. Show the following:

- (i) p divides $\binom{p}{k}$.
- (ii) $(a+b)^p \equiv a^p + b^p \pmod{p}$.