

## Math 311 Review for Exam 1 Fall 09

1. Use mathematical induction to show the following:

(a) 
$$\sum_{k=1}^n k(k!) = (n+1)! - 1.$$

(b) If  $a > -1$ , then  $(1+a)^n \geq 1+an$ .

2. Use the Binomial Theorem to verify 
$$\sum_{k=0}^n \binom{n}{k} (-2)^k = (-1)^n.$$

3. Prove the following:

If  $\gcd(a, b) = 1$  and  $a|bc$  then  $a|c$ .

4. Find the greatest common divisor of 1008 and 1625. Then express the  $\gcd(1008; 1625)$  in the form  $1008x + 1625y$ . Show all your work.

5. Find all integers  $a$  for which  $a^2 + (a+2)^2 + (a+4)^2 + 1$  is divisible by 12.

6. (a) Show that  $357x + 135y = 111,111,110$  has no integer solution.

(b) Find all  $m$  for which  $49x + 77y = m$  has integer solutions. Justify your answer.

(c) Determine **all** solutions to the equation  $49x + 77y = 105$ .

7. Let  $r$  be positive integer greater than 1 with the following property. *For any pair of integers  $a$  and  $b$  the following holds: If  $r$  divides  $ab$  then  $r$  divides  $a$  or  $r$  divides  $b$ .*

Show that  $r$  is a prime.

8. A man has \$4.55 in change composed entirely of dimes and quarters. What are the maximum and minimum number of coins that he can have?

9. (a) Given that  $p$  is a prime, show that  $\sqrt{p}$  is irrational.

(b) Find the highest power of 53 that divides  $100!$ . Explain your reasoning.

10. Review homework assignments.