Math 311 Review for Exam 2 Fall 09

1. Find all incongruent solutions to the following (if they exist):
(a) $7x \equiv 5 \pmod{12}$. (b) $9x \equiv 6 \pmod{39}$. (c) $75x \equiv 15 \pmod{225}$.

2. Page 68, Problem 6.

3. Page 82, Problem 4.

4. Page 83, Problem 10.

5. Page 92, Problem 7.

6. (a) Show that 17 divides $28^{104} + 1$.
(b) Show that $a^{13} \equiv a \pmod{273}$ for all integers a.
(c) Page 97, Problem 15.

7. Find the remainder when 21! is divided by 23.

8. (a) Let $f(n)$ be a multiplicative function. Show that $F(n) = \sum_{d|n} f(d)$ is also multiplicative.
(b) Show that $\sigma(n) = \sum_{d|n} d$ is multiplicative.

9. (a) Find $\sigma(23716)$, find $\tau(23716)$.
(b) Let p be a prime and k a positive integer. Find $\sigma(p^k)$ and $\tau(p^k)$.

10. (a) Page 110, Problem 8.
(b) Page 117, Problem 1.

11. Prove the following:
(a) If $a \equiv b \pmod{n}$, prove that $gcd(a, n) = gcd(b, n)$.
(b) Assume that $gcd(m, n) = 1$. Show that if m divides a and n divides a then mn divides a.
(c) Assume that $gcd(m, n) = 1$. Show that $a \equiv b \bmod m$ and $a \equiv b \bmod n$ implies that $a \equiv b \bmod mn$.
(d) Assume that $gcd(m, a) = 1$. Show that, if m divides ab, then m divides b.

12. Page 135, Problem 4.

Old homeworks. Congruences, Chinese Remainder Theorem, Fermat's Little Theorem, multiplicative functions, σ and τ, Möbius Inversion formula, Euler's ϕ-function.