Math 518 Spring 2003 Homework 5 due March 26:

1. Let $T \in \mathcal{L}(\mathbb{C}^3)$ with $T(x, y, z) = (y, z, 0)$. Find all eigenvalues of $T$. Find a basis for $\mathbb{C}^3$ that consists entirely of generalized eigenvectors and find the corresponding matrix.

2. Let $T \in \mathcal{L}(V)$, $m$ a positive integer, $v \in V$ such that $T^{m-1}(v) \neq 0$ but $T^m(v) = 0$. Prove that $\{v, T(v), T^2(v), ..., T^{m-1}(v)\}$ is linearly independent.

3. Let $T \in \mathcal{L}(V)$ such that $T^m = 0$ for some integer $m$. Show that the only eigenvalue of $T$ is 0.

4. Let $\mathbf{P}_4(\mathbb{R})$ denote the space of all polynomials in $x$ of degree less than or equal to 4 with coefficients in $\mathbb{R}$. Define the linear map $D : \mathbf{P}_4(\mathbb{R}) \to \mathbf{P}_4(\mathbb{R})$ as $D(p(x)) = p'(x)$. Find all eigenvalues of $D$ then choose a basis $B$ of generalized eigenvectors and find the matrix of $D$ corresponding to $B$.

5. Let $V$ be a finite dimensional vector space and $T \in \mathcal{L}(V, V)$. Suppose that $\text{range}(T) = \text{range}(T^2)$. Prove that $V = \text{range}(T) \oplus \text{null}(T)$.

6. Number 4 on page 94.

7. Number 7 on page 94.

8. Number 9 on page 94.