

Finitely generated ideals in rings of integer-valued polynomials

Let D be an integral domain with field of fractions Q , and E be a finite non-empty subset of D ; we set $\text{Int}(E,D) = \{f(X) \in Q[X] : f(E) \text{ is a subset of } D\}$, the ring of integer-valued polynomials on D with respect to the subset E . Recall that a Prufer domain is an integral domain in which every non-zero finitely generated ideal is invertible. It is known that D is a Prufer domain if and only if $\text{Int}(E,D)$ is a Prufer domain, so that the $\text{Int}(E,D)$ construction yields a method for producing new Prufer domains from old. In this talk, we determine the relationship between the minimum number of generators needed for finitely generated ideals of D to that of $\text{Int}(E,D)$. As a corollary, we show that iterating the $\text{Int}(E,D)$ construction cannot produce a sequence of Prufer domains whose finitely generated ideals require an ever larger number of generators.