The doubly transitive permutation representations of $\mathrm{S}p(2n, 2)$

The Symplectic groups over the field of two elements each have two doubly transitive actions. These actions were discovered in the 19th century and appear in the works of Steiner, Jordan and Riemann. There are many ways to think of these actions; as an affine action on points of a quadric; as the action on sets of quadratic forms;or as the action on the "theta characteristics" of an algebraic curve of genus n. In this talk, we will describe the rich geometric algebra of these group actions and consider the structure of the mod 2 permutation module. In particular we describe filtrations of these modules such that the subquotients have characters which are given by Weyl's Character Formula from Lie theory. The first half of this talk requires only knowledge of linear algebra and basic group theory. Finally, we will discuss the connections with curves and, if time permits, coding theory.