

## Math 237.101 Exam 2 Information

**Exam 2** will be on **Wednesday, November 9.**

It will cover Sections 3.1-3.4, and 4.1-4.3.

No calculators, books, or notes, except for one 3" by 5" note card, are allowed on the test.

The exam will contain 10 True/False questions and a number of problems where calculations and/or explanations are required. The problems on the test will be similar to homework problems, so go over the Hw problems as needed. Also, review the definitions and properties discussed in the course.

A complete list of Hw assignments is available at the course home page

[http://www.southalabama.edu/mathstat/personal\\_pages/sadovska/237/237.html](http://www.southalabama.edu/mathstat/personal_pages/sadovska/237/237.html)

In addition, answer the following questions:

1.  $A$  is an  $n \times n$  matrix. Give at least five conditions equivalent to the condition  $A$  is invertible.
2. Complete the statement: If  $A$  is diagonal or triangular, then its determinant is \_\_\_\_\_, and its eigenvalues are \_\_\_\_\_.
3. Describe two ways to find  $\det A$ .  
How do elementary row operations affect the determinant?
4.  $A$  and  $B$  are invertible  $n \times n$  matrices with  $\det A = a$  and  $\det B = b$ .  
Find  $\det A^T$ ,  $\det A^{-1}$ ,  $\det(-A)$ ,  $\det(kA)$ ,  $\det A^m$ ,  $AC^T$  ( $C$  is the matrix of cofactors),  $\det C$ ,  $\det AB$ , and  $\det(A^T B^{-1} A^2)$ .
5. What is the definition of an eigenvalue? eigenvector? eigenspace?  
How to find the characteristic polynomial, eigenvalues, and the corresponding eigenspaces of a matrix?
6. Similar matrices have the same \_\_\_\_\_, the same \_\_\_\_\_, and the same \_\_\_\_\_.
7. How to find out if a square matrix is diagonalizable, and if it is, how to find an invertible matrix  $P$  and a diagonal matrix  $D$  such that  $P^{-1}AP = D$ ?  
Which condition(s) guarantee that  $A$  is diagonalizable?
8. Multiplication by an  $m \times n$  matrix defines a function from \_\_\_\_\_ to \_\_\_\_\_.
9. How to find the null space of a matrix?  
If  $A$  is square, which condition(s) guarantee a trivial null space?  
If  $A$  is  $m \times n$ , which condition guarantees a non-trivial null space?

10. How to tell if a vector is in the null space of a matrix? in its column space?
11. How to find the rank of a matrix?  
What does the rank tell us about the number of solutions of  $A\mathbf{x} = \mathbf{b}$ ?
12. A vector  $\mathbf{b}$  is in the span of  $\mathbf{u}_1, \dots, \mathbf{u}_n \Leftrightarrow \underline{\hspace{2cm}} \Leftrightarrow \underline{\hspace{2cm}}$ .  
What can you say about the span of vectors?
13. Give at least 3 examples of vector spaces other than  $\mathbb{R}^n$ .
14. What is the definition of a subspace of a vector space?  
Which conditions one needs to check to find out if a set  $U$  is a subspace of a vector space?
15. Describe all subspaces of  $\mathbb{R}^2$  and all subspaces of  $\mathbb{R}^3$ .
16. How to test vectors for linear dependence/independence?  
When is it easy to tell that vectors are LD?
17. Let  $V$  be an  $n$ -dimensional vector space.  
What can you say about the number of vectors in a LI set in  $V$ ? in a spanning set?
18. What is a basis of a vector space?
19. If  $n$  vectors are given, how to tell if they form a basis of  $\mathbb{R}^n$ ?
20. If several vectors in  $\mathbb{R}^n$  are given, how to find a basis and the dimension of their span?
21. How to find a basis and the dimension of the null space of  $A$ ?  
of the column space of  $A$ ? of the row space of  $A$ ?
22. What can you say about the rank and the row rank of  $A$ ?  
If  $A$  is  $m \times n$ , what is  $(\text{rank } A + \text{nullity of } A)$ ?