Math 316 Homework 1

due Wednesday, 01/18

1. $\mathbf{v_1}$, ..., $\mathbf{v_k}$, and $\mathbf{u}$ are vectors. Complete the sentence:

Span $\{\mathbf{v_1}, \ldots, \mathbf{v_k}\} =$ *Span* $\{\mathbf{u}, \mathbf{v_1}, \ldots, \mathbf{v_k}\}$ *if and only if* $\mathbf{u}$

2. Let U be the set of all vectors in $\mathbb{R}^3$ of the form $\begin{bmatrix} 0 \\ x \\ x^2 \end{bmatrix}$.

Is U a subspace of $\mathbb{R}^3$? *Justify your answer.*

3. Let A be an $n \times n$ matrix, let λ be a number, and let U be the set of vectors $\mathbf{x}$ in $\mathbb{R}^n$ such that $A\mathbf{x} = \lambda\mathbf{x}$. Prove that U is a subspace of $\mathbb{R}^n$.

Note that U always contains $\mathbf{0}$. If $U \neq \{\mathbf{0}\}$, then U is the eigenspace corresponding to the eigenvalue λ.

4. Let $\mathbf{v_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \ldots \\ 0 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ \ldots \\ 0 \end{bmatrix}$, ..., $\mathbf{v_{n-1}} = \begin{bmatrix} 1 \\ 1 \\ \ldots \\ 1 \\ 0 \end{bmatrix}$, and $\mathbf{v_n} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \ldots \\ 1 \end{bmatrix}$.

For any vector $\mathbf{x} = \begin{bmatrix} x_1 \\ \ldots \\ x_n \end{bmatrix}$, find coefficients c_1, ..., c_n such that $\mathbf{x} = c_1\mathbf{v_1} + \ldots + c_n\mathbf{v_n}$.

Note: This is a way to show that $\mathbf{v_1}$, ..., $\mathbf{v_n}$ is a spanning set in $\mathbb{R}^n$.

5. Prove that the matrices E_{11}, E_{12}, ..., E_{23} form a basis of the space of all 2×3 matrices.