Math 316   Homework 12 due Wednesday, 4/26

If you would like to get your Hw back on Wednesday, 4/26, submit it on Monday or leave it in my mailbox before 5 p.m. on Tuesday.

1. Let $J = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix}$

   Note: No calculations are needed in this problem.

   (a) What is the characteristic polynomial of J?

   (b) What are the eigenvalues and the corresponding eigenvectors of J?

   (c) For $k = 1, \ldots, 7$, find Je_k and $(J - \lambda I)e_k$ for appropriate λ.

   (d) Represent $\mathbb{R}^7$ as a direct sum of subspaces invariant under multiplication by J.

2. (a) Find the Jordan canonical forms of the matrices

   $$A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & -4 & 3 \end{bmatrix}, C = \begin{bmatrix} 3 & -1 & 2 \\ 4 & -1 & 3 \\ 0 & 0 & 1 \end{bmatrix}, \text{ and } F = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

   (b) Which of the matrices B, C, and F are similar?

3. Give an example of two 5 by 5 matrices which have characteristic polynomial $-\lambda^5$ and 3 LI eigenvectors each, but are not similar.

4. We know that two matrices are similar if and only if they have the same Jordan canonical form. Give a simpler condition for diagonalizable matrices:

   Two diagonalizable matrices are similar if and only if

Math 316 Final Exam information

The exam will be on **Wednesday, May 3, 3:30-5:30 p.m.** in ILB 345

The following topics will be emphasized:

- Complex matrices, in particular Hermitian and unitary matrices (7.2).
- Diagonalization (3.4).
- Unitary diagonalization, Spectral Theorem, normal matrices (7.3).
- Jordan canonical form.

The test will also cover the following topics:

- Linear transformations 5.1 Ex. 1, 7.
- Coordinates of a vector, matrices of a linear transformation. 5.3 Ex. 2, 10.
- Changing coordinates. 5.4 Ex. 1, 3(a), 8, 9, 13.
- Orthogonal and orthonormal sets and bases, Gram-Schmidt algorithm.
  6.2 Ex. 1(b,c); 7.1 Ex.16.
  Note: You should be able to apply the G-S algorithm to complex vectors as well.
- Complex vectors. 7.1 Ex. 11, 12, 15.
- A theorem of Shur - statement only. 7.3 Ex. 19.

The exam will contain short-answer questions, problems where proofs or explanations are required, and computational problems. Most of the problems on the test will be similar to homework problems (so go over these problems as needed). It is also important to know the definitions and properties discussed in the course. No books, notes, or calculators are allowed on the test.