MA 334 STUDY GUIDE FOR EXAM I

Exam 1 will be on **Wednesday, September 26.**
No books, notes, or calculators are allowed on the test.

You will be asked to
- State definitions and theorems.
- Answer true/false questions.
- Give proofs. These problems will be similar to homework problems.
  Make sure that you know how to solve all Hw problems.

Here is a list of definitions and theorems that you need to know.

Definitions.

1. Give definitions of the intersection, union, and Cartesian product of two sets.
2. f is a function $S \to T$. What is the domain of f? the range of f? What does it mean that f is onto? one-to one?
3. What does mean that a sequence converges to a?
4. What does mean that a sequence does not converge to a?
5. What does it mean that a sequence diverges to ∞? to $-\infty$?
6. What does mean that a sequence is bounded?
7. What does mean that a sequence is Cauchy?
8. What does mean that a sequence is increasing? decreasing?

Theorems.

1. Triangle inequality.
2. Limit of a convergent sequence is unique.
3. If a sequence converges, then it is bounded.
4. Limit theorems for sum, multiple, product and quotient of two sequences.
5. A sequence of real numbers is a Cauchy sequence if and only if it converges to a (finite) real number.
6. Every bounded monotone sequence converges.

True/False questions. You should determine whether or not a statement is true. If a statement is false, you must provide a counter example. The following two examples show how to answer questions of this type.

(a) A convergent sequence has a unique limit. Answer: True

(b) If $\{a_n + b_n\}$ converges, then both $\{a_n\}$ and $\{b_n\}$ converge. Answer: False.
If $a_n = n$ and $b_n = -n$, then $\{a_n + b_n\} = \{0\}$ converges, but $\{a_n\}$ and $\{b_n\}$ diverge.

1. For any real numbers x and y, $|x + y| \leq |x| + |y|$.
2. For any real numbers x and y, $|x - y| \leq |x| - |y|$.
3. Every convergent sequence is bounded.
4. Every bounded sequence converges.
5. Every bounded monotone sequence converges.
6. Every monotone sequence converges.
7. Every convergent sequence is monotone.
8. A divergent sequence must diverge to ∞ or $-\infty$.
9. If $a_n \to 1$, then $.9 < a_n < 1.1$ for all n except for finitely many.
10. If $a_n \leq b_n$ for all n and $b_n \to 0$, then $a_n \to 0$.
11. If $\{a_n\}$ is non-negative, $a_n \leq b_n$ for all n, and $b_n \to 0$, then $a_n \to 0$.
12. If $a_n < b$ for all n and $\{a_n\}$ converges, then $\lim_{n\to\infty} a_n < b$.
13. If a sequence $\{(a_n)^2\}$ converges, then the sequence $\{a_n\}$ also converges.
14. Product of two convergent sequences converges.
15. Quotient of two convergent sequences converges.
16. If $\{a_n + b_n\}$ and $\{a_n\}$ converge, then $\{b_n\}$ also converges.
17. If $\{a_n b_n\}$ and $\{a_n\}$ converge, then $\{b_n\}$ also converges.
18. If $\{a_n\}$ converges, then $\{1/a_n\}$ diverges.
19. If $a_n \to 0$, then $1/a_n \to \infty$.
20. Every Cauchy sequence is bounded.
21. Every Cauchy sequence of real numbers converges to a real number.
22. Every Cauchy sequence of rational numbers converges to a rational number.
23. Every convergent sequence is a Cauchy sequence.
24. If a sequence $\{a_n\}$ is monotone and bounded, then the sequence $\{b_n\}$, where $b_n = (a_1 + \ldots + a_n)/n$, is also monotone and bounded.